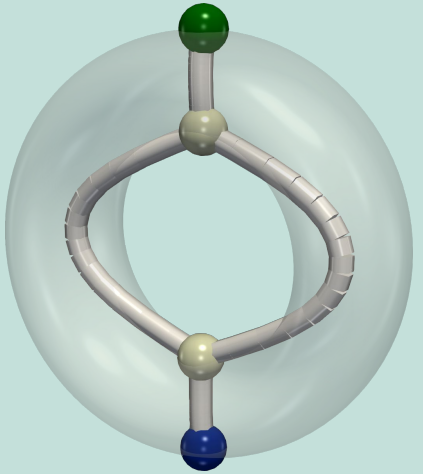
Topological Analysis of Massive Datasets



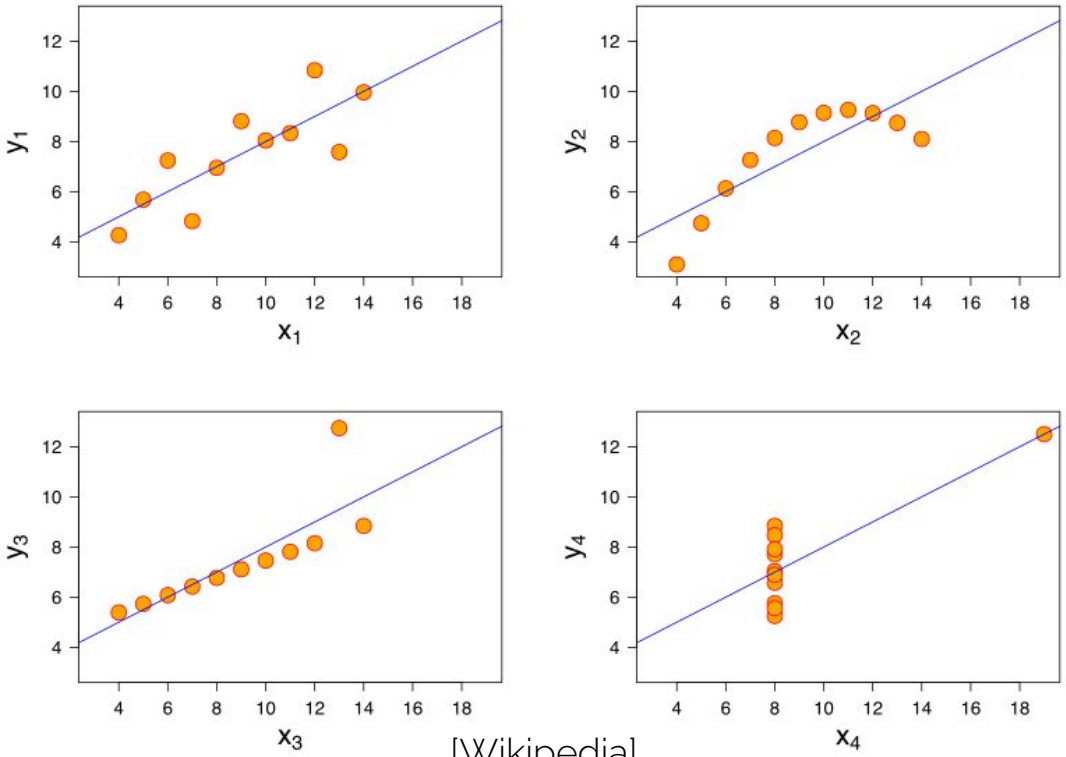
Julien Tierny

cnrs SORBONNE UNIVERSITÉ CRÉATEURS DE FUTURS DEPUIS 1257 LIP6 erc

Capturing the *“shape”* of data

Capturing the *"shape"* of data

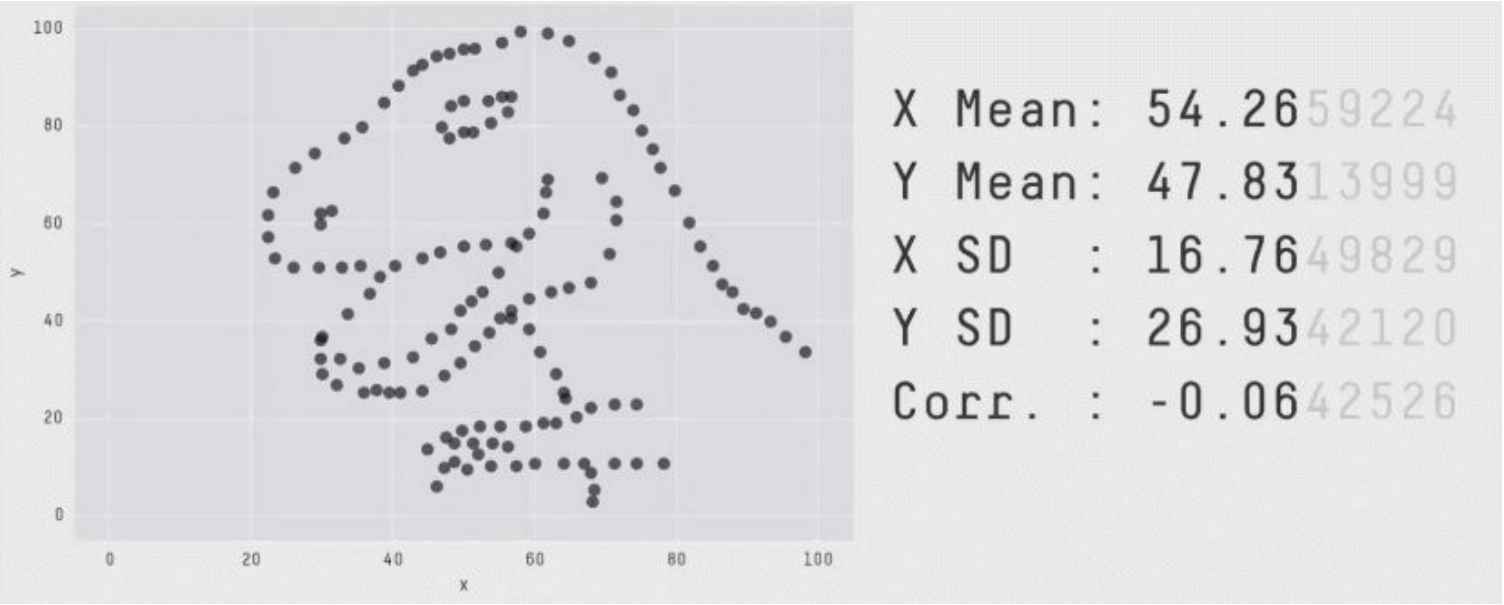
- Anscombe's quartet



[Wikipedia]

Capturing the *“shape”* of data

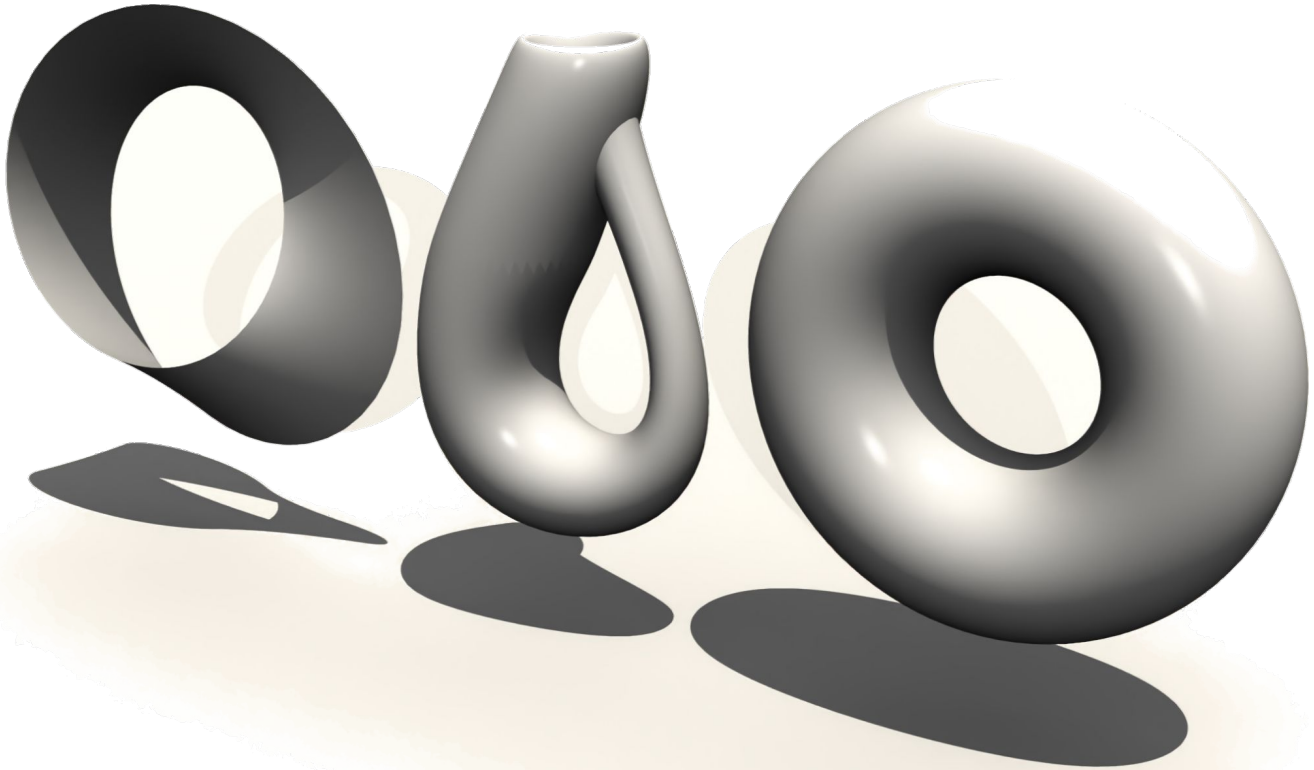
- Datasaurus Dozen



[Matejka et al. 2017]

Capturing the *"shape"* of data

- Topological invariants



The topology of numerical data

	Row ID	x	y
0	0	-0.565872	0.433185
1	1	-0.852855	-0.0414391
2	2	-0.74526	-0.0704809
3	3	-0.283754	0.732374
4	4	-0.348494	0.708251
5	5	-0.141337	-0.791217
6	6	-0.590461	0.429422
7	7	-0.611657	-0.551685
8	8	-0.601515	-0.45884
9	9	0.240081	0.697199
10	10	0.101971	0.836754
11	11	-0.706277	-0.36127
12	12	0.301446	-0.731123
13	13	-0.759347	0.297477
14	14	-0.685959	0.140645
15	15	-0.220021	-0.828495
16	16	0.794982	0.352219
17	17	0.61109	0.561273
18	18	-0.642173	0.313909
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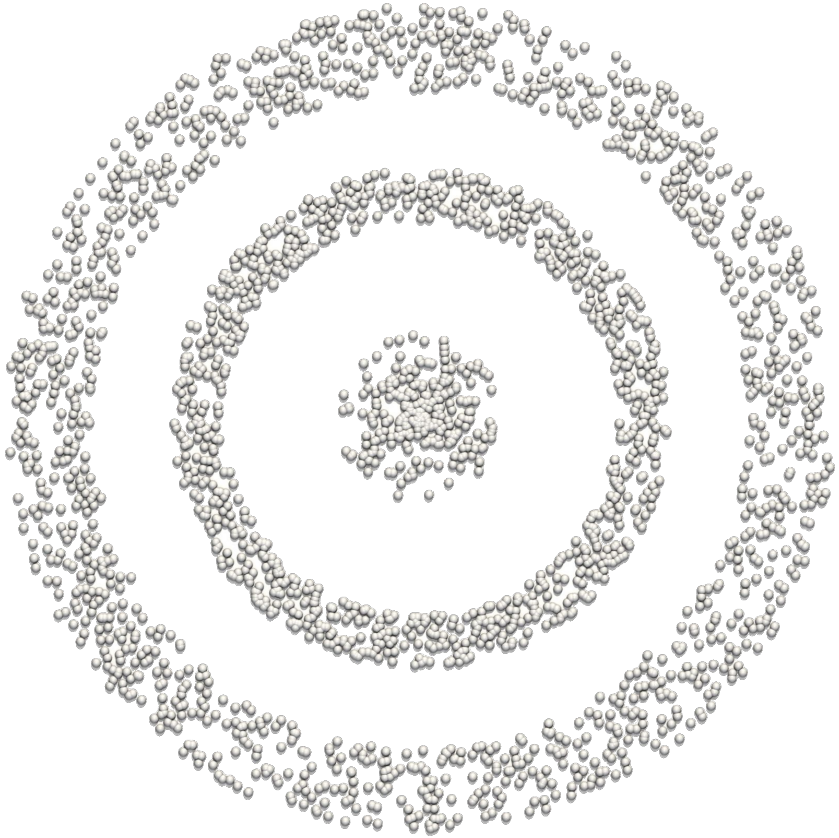

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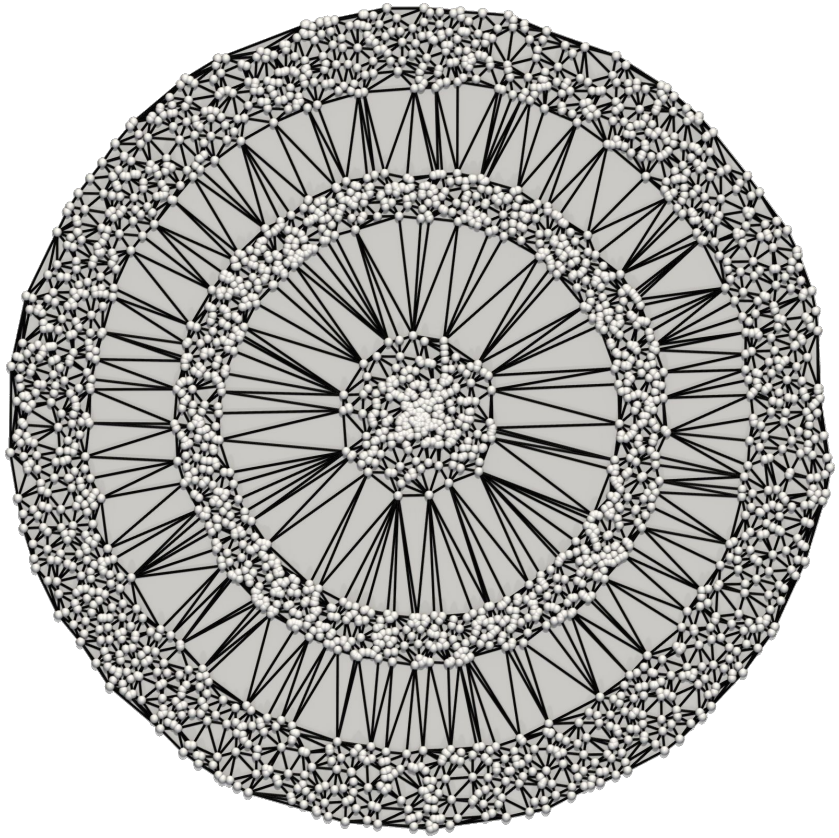

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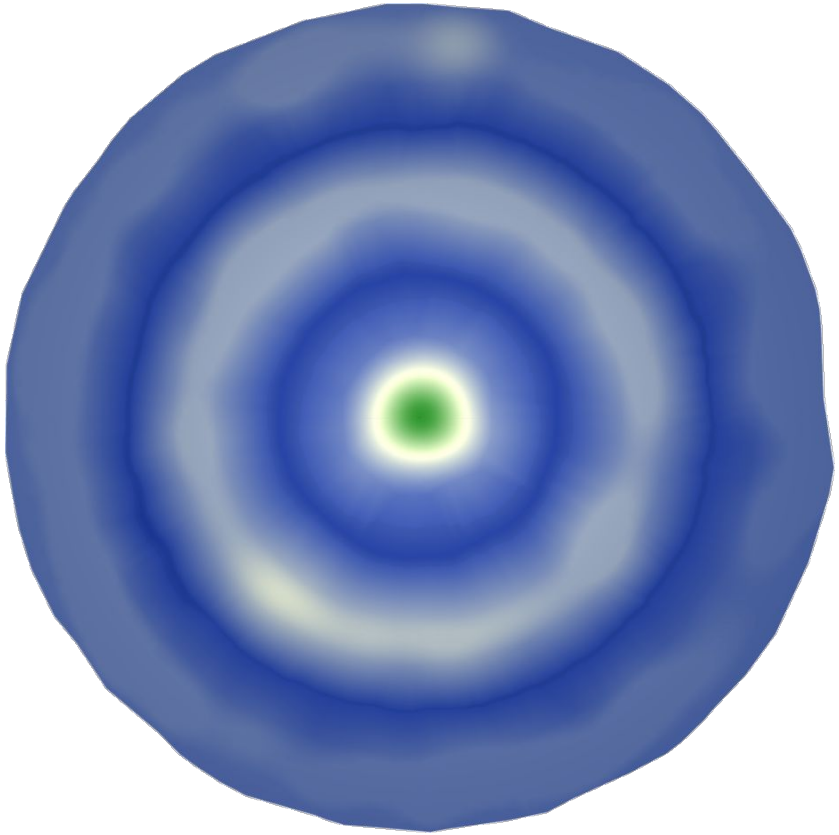

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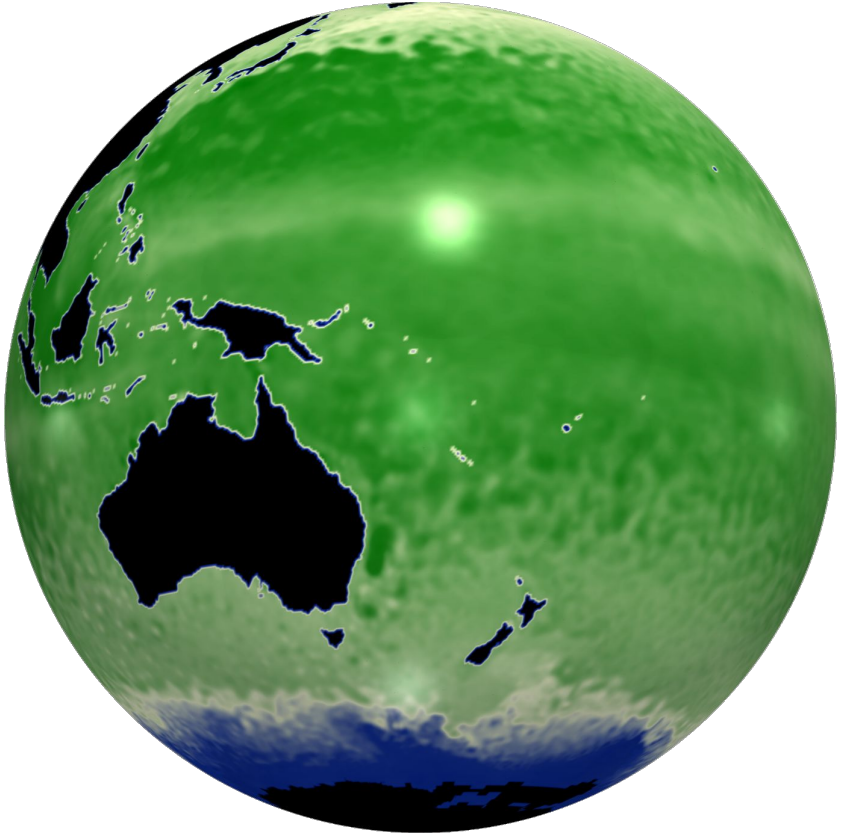

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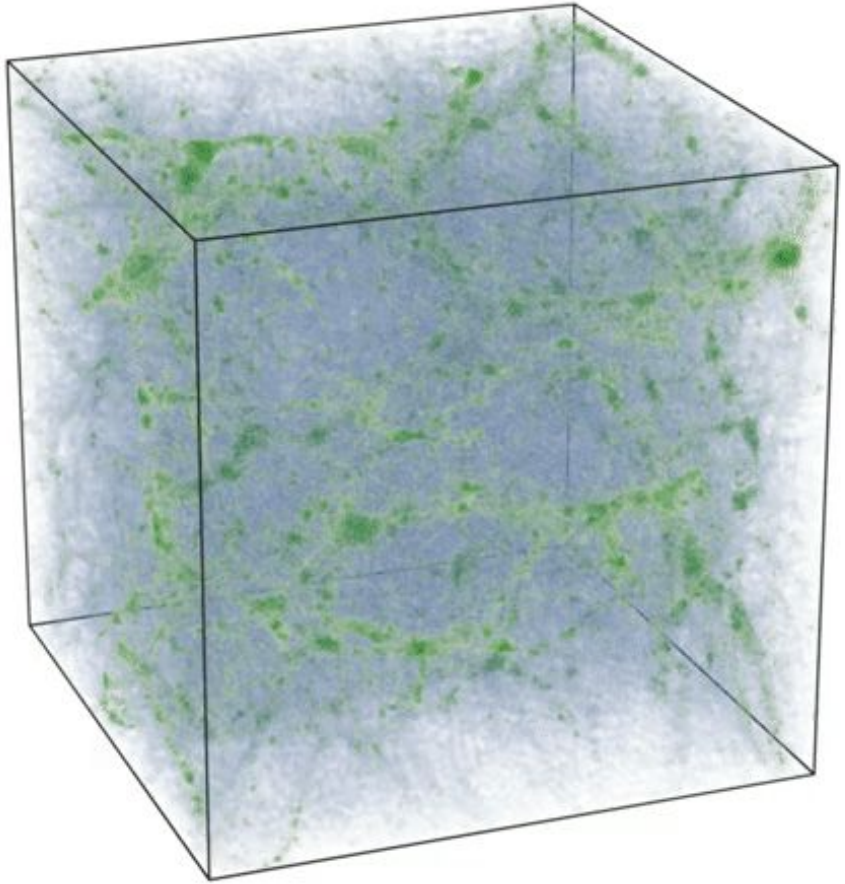

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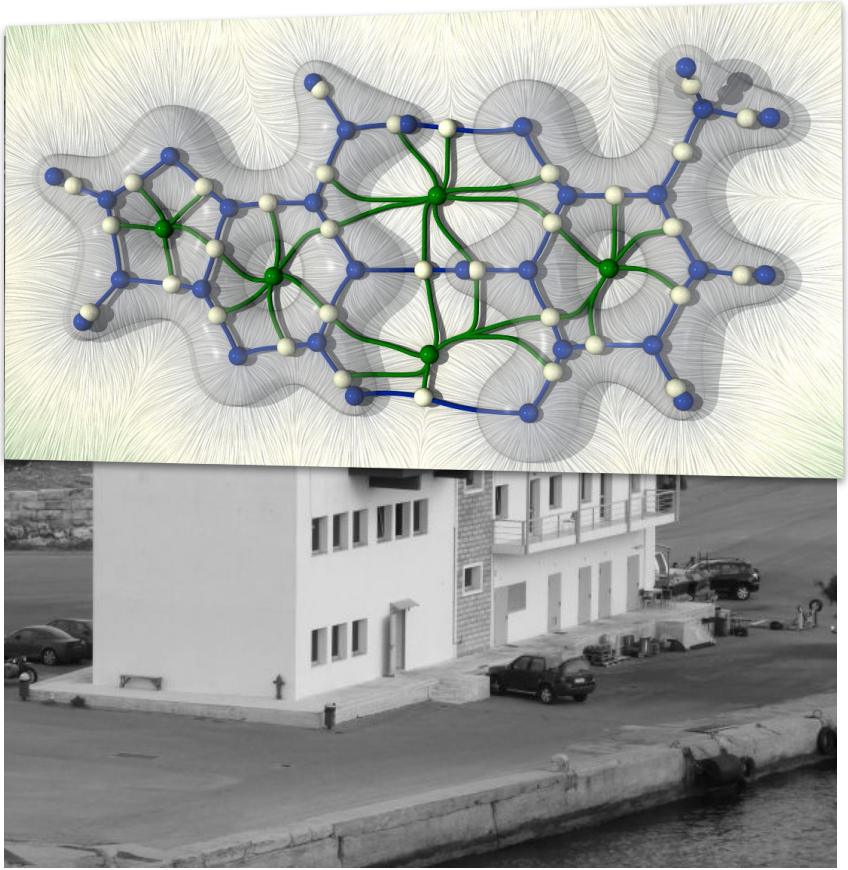

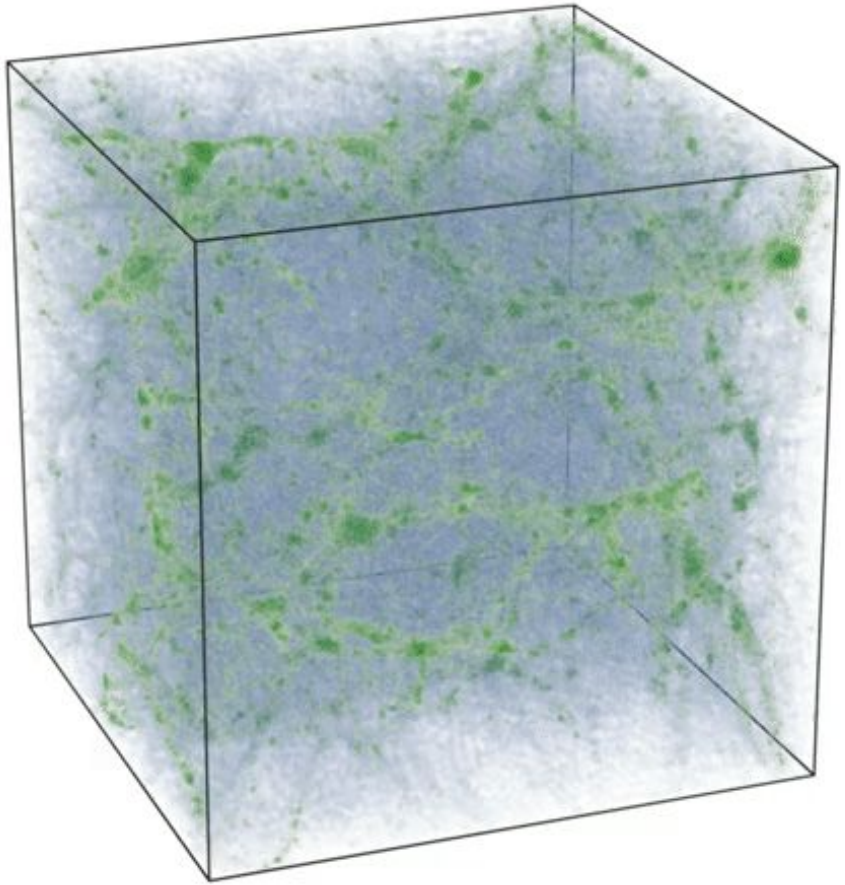

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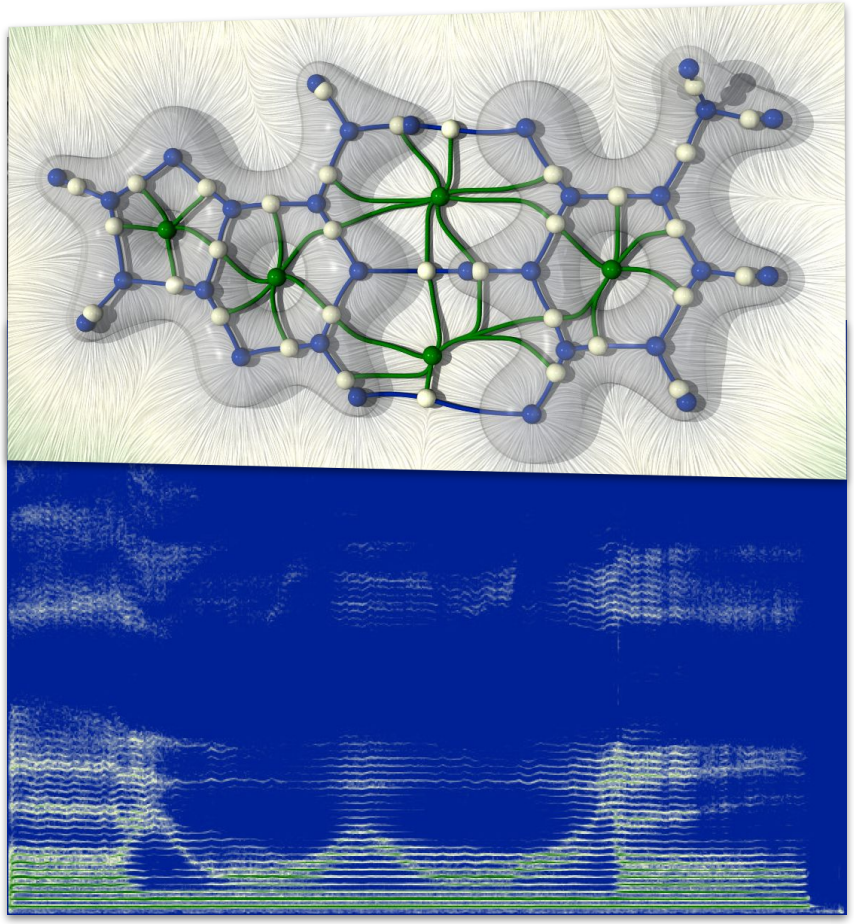

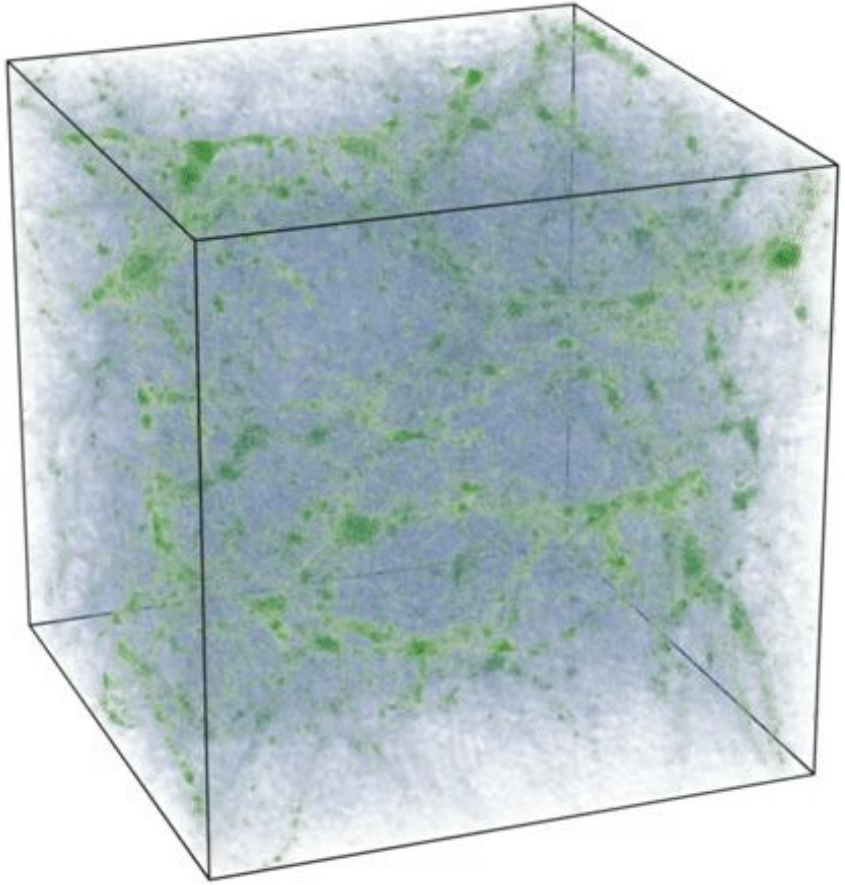

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- Input PL scalar data
  - $f : \mathcal{M} \to \mathbb{R}$

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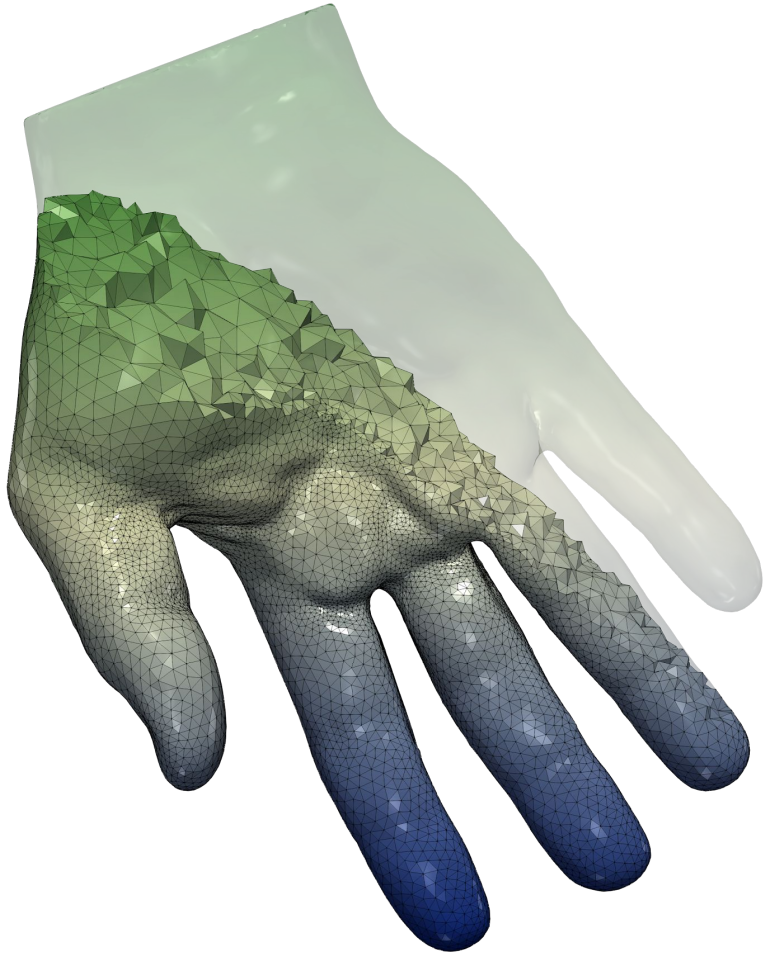
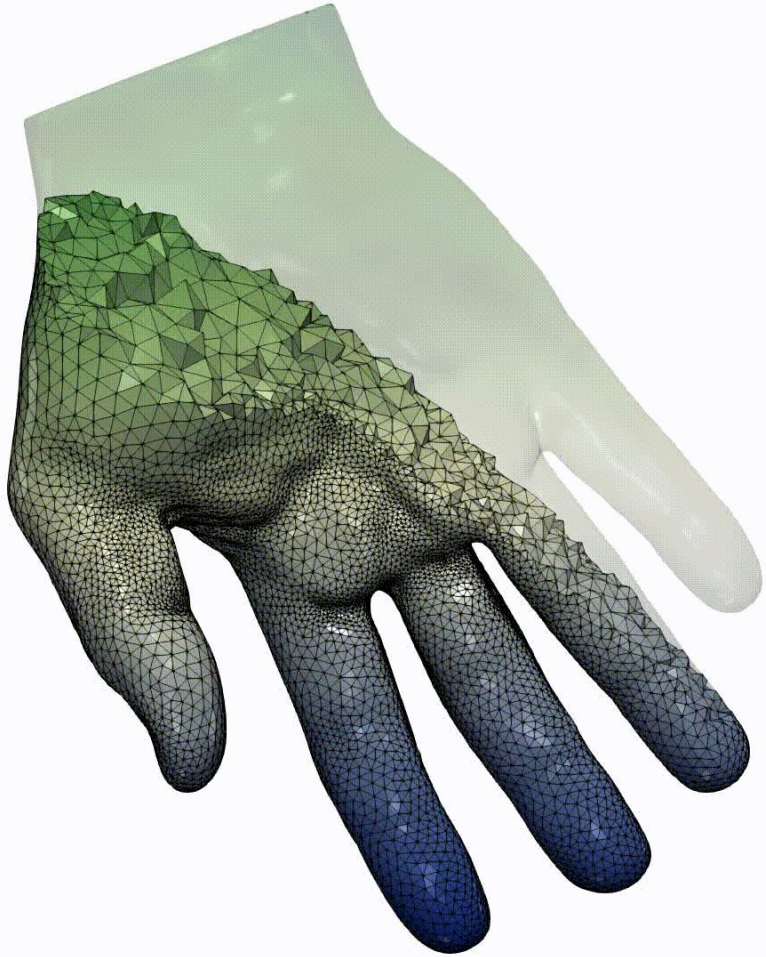


How does it work?

- **Input PL scalar data**
  - $f : \mathcal{M} \to \mathbb{R}$
  - $\mathcal{M}$: simplicial complex
    - Triangulated surface
    - Tetrahedral volume
    - 2D pixel image
    - 3D voxel image
    - Point cloud data



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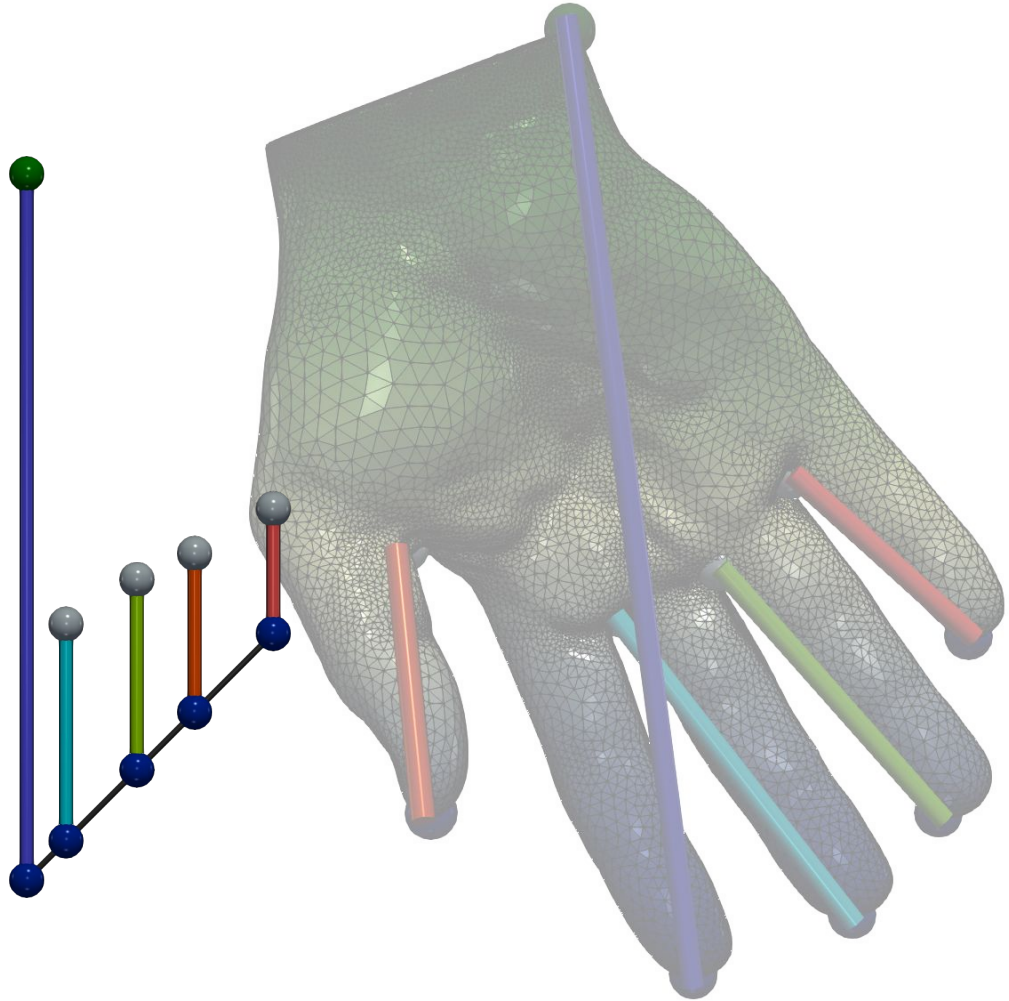
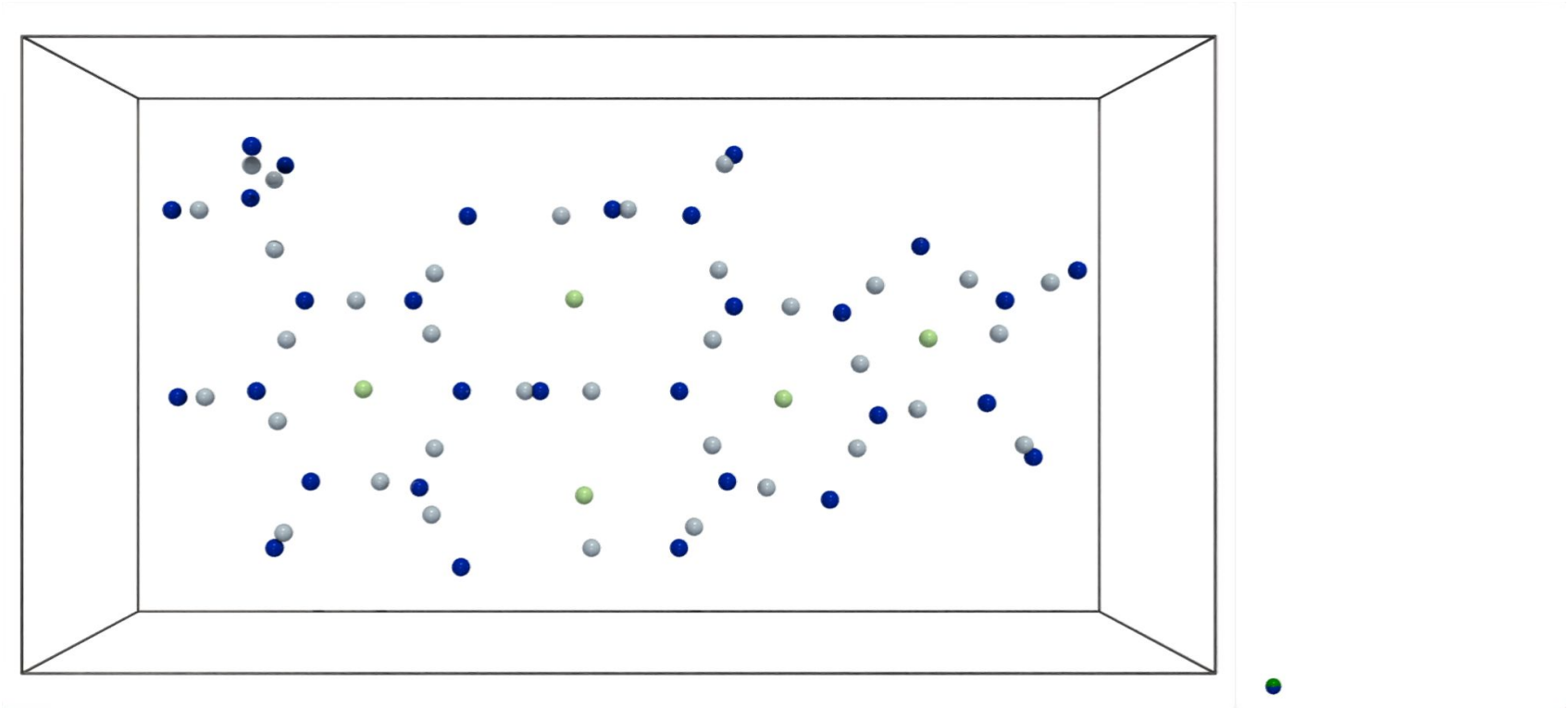
  - Persistence diagram



The persistence diagram

- Beyond connected components



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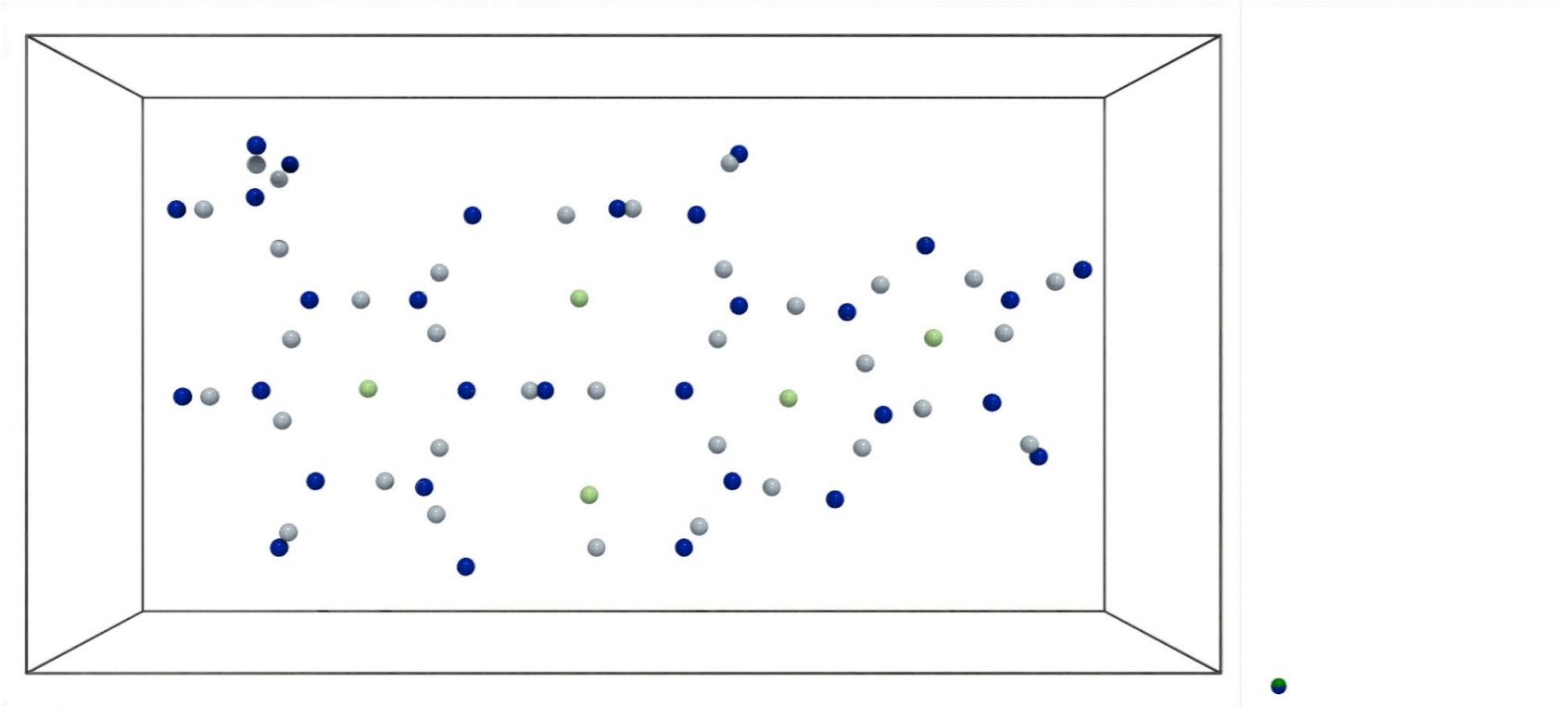

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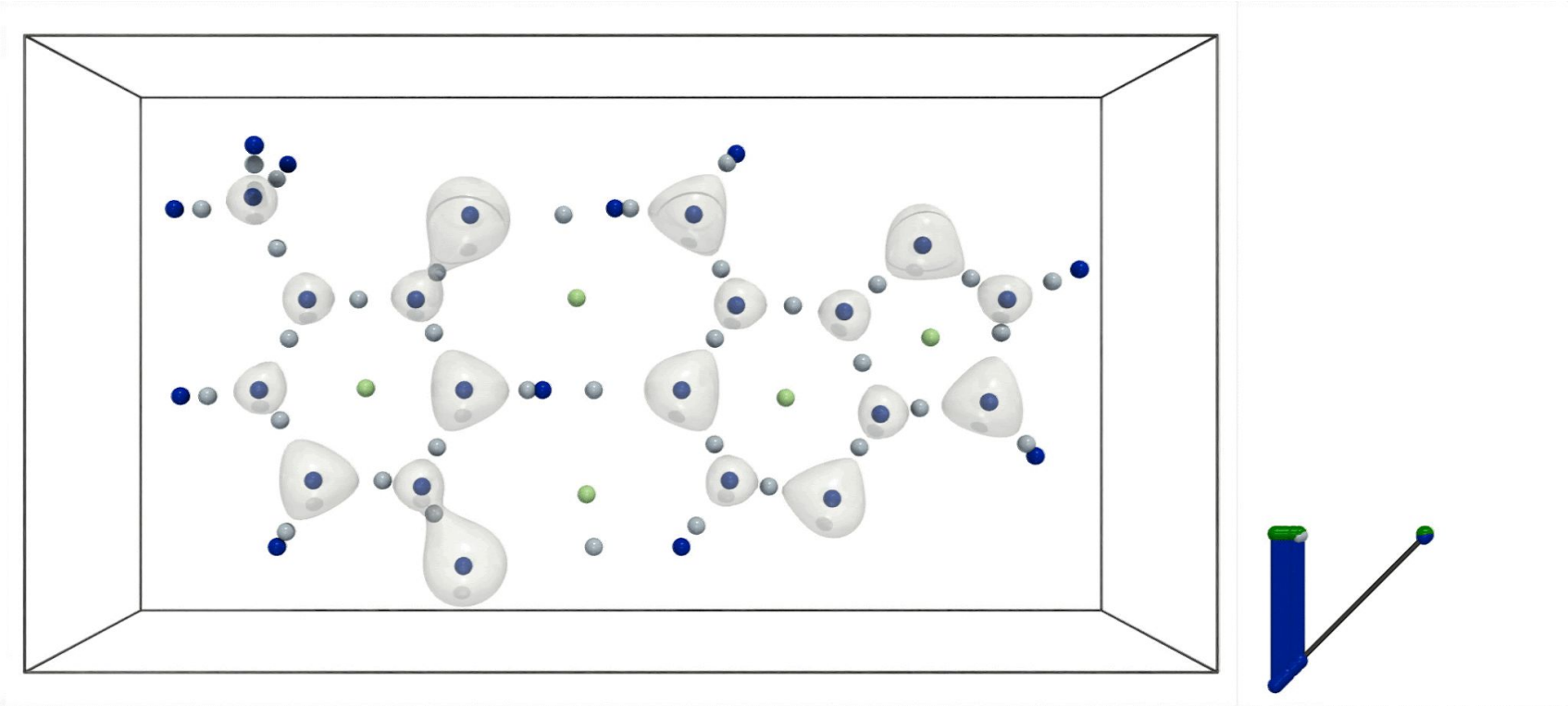

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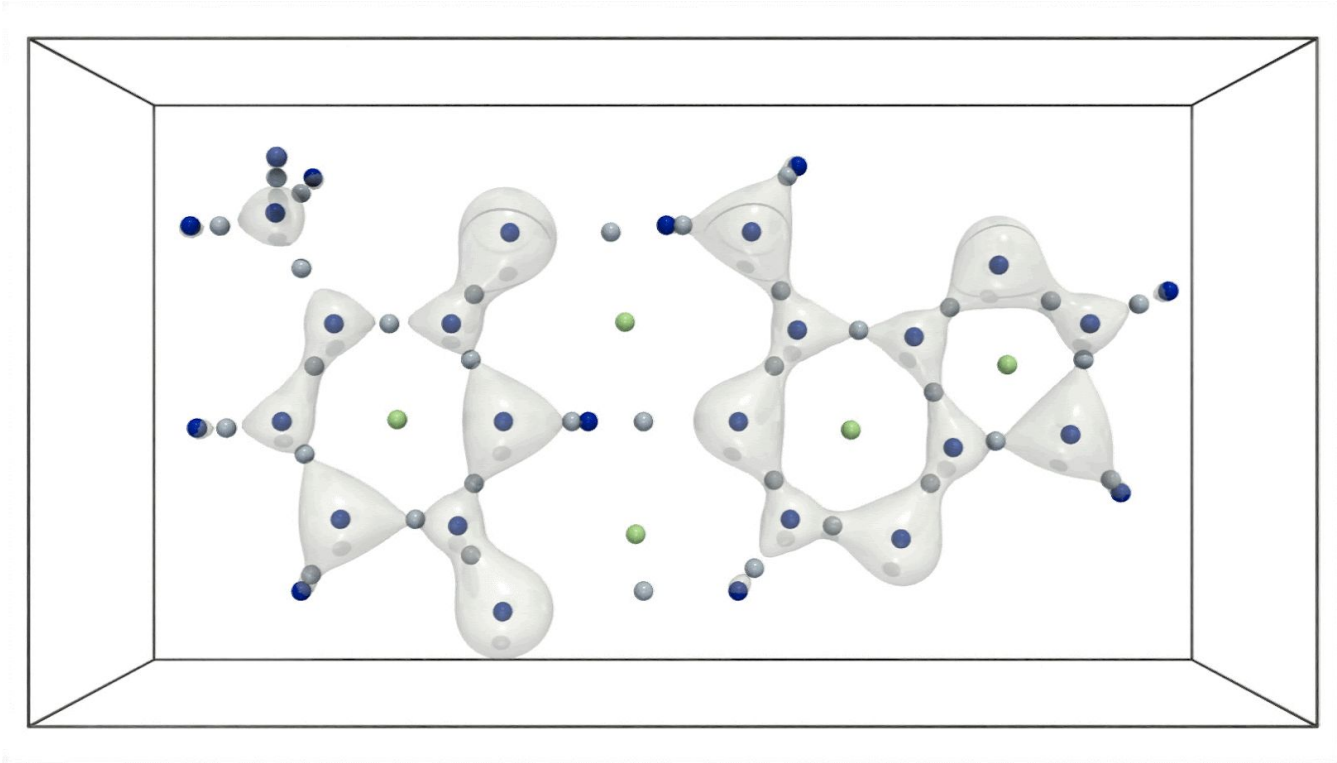

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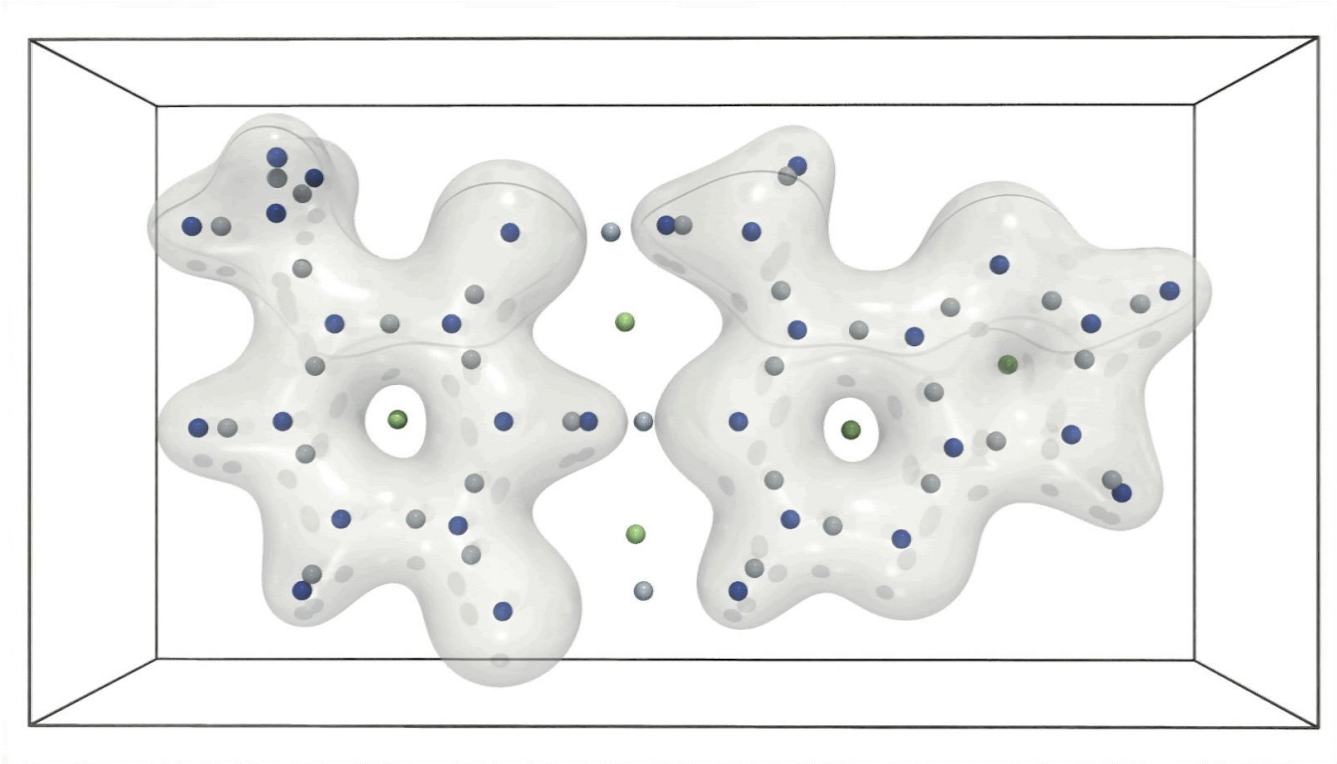

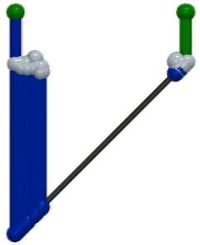

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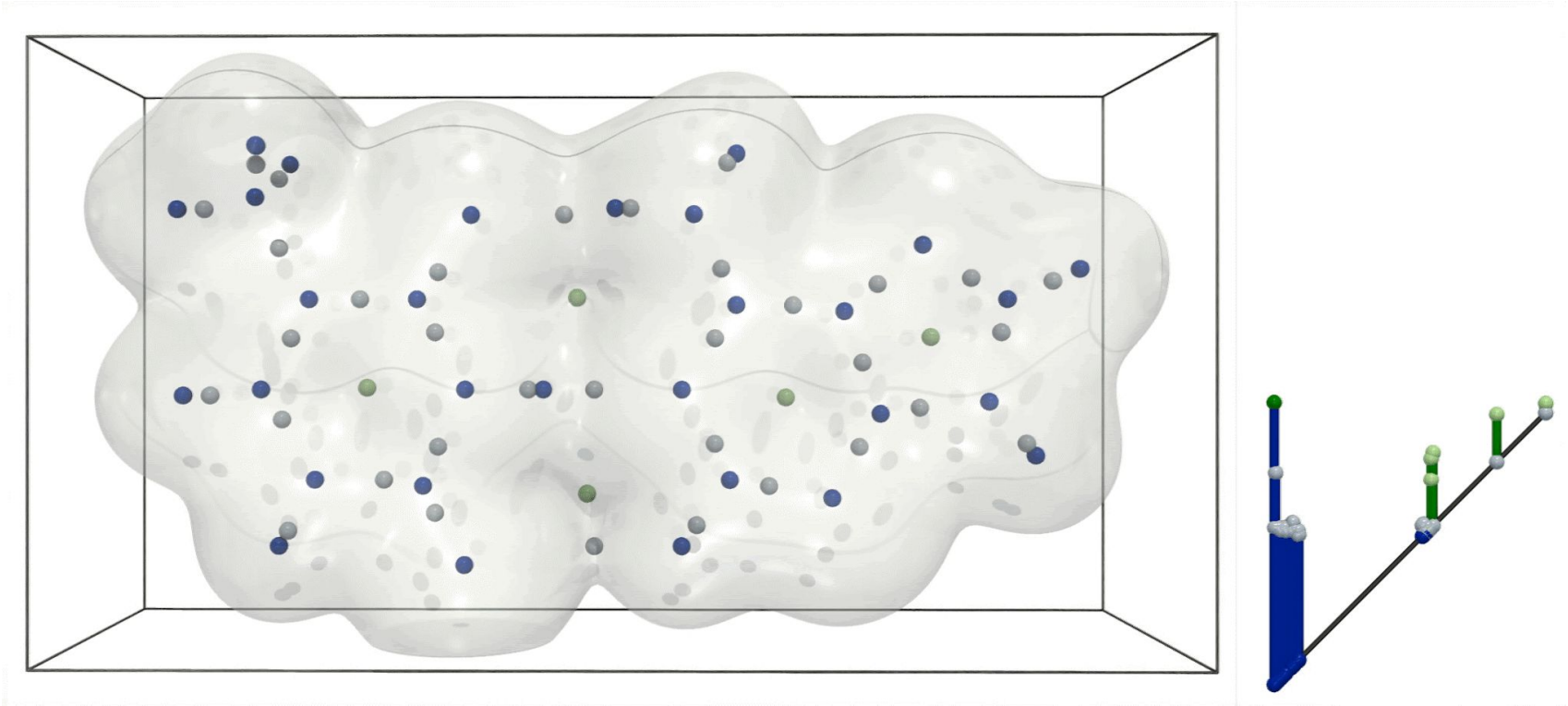

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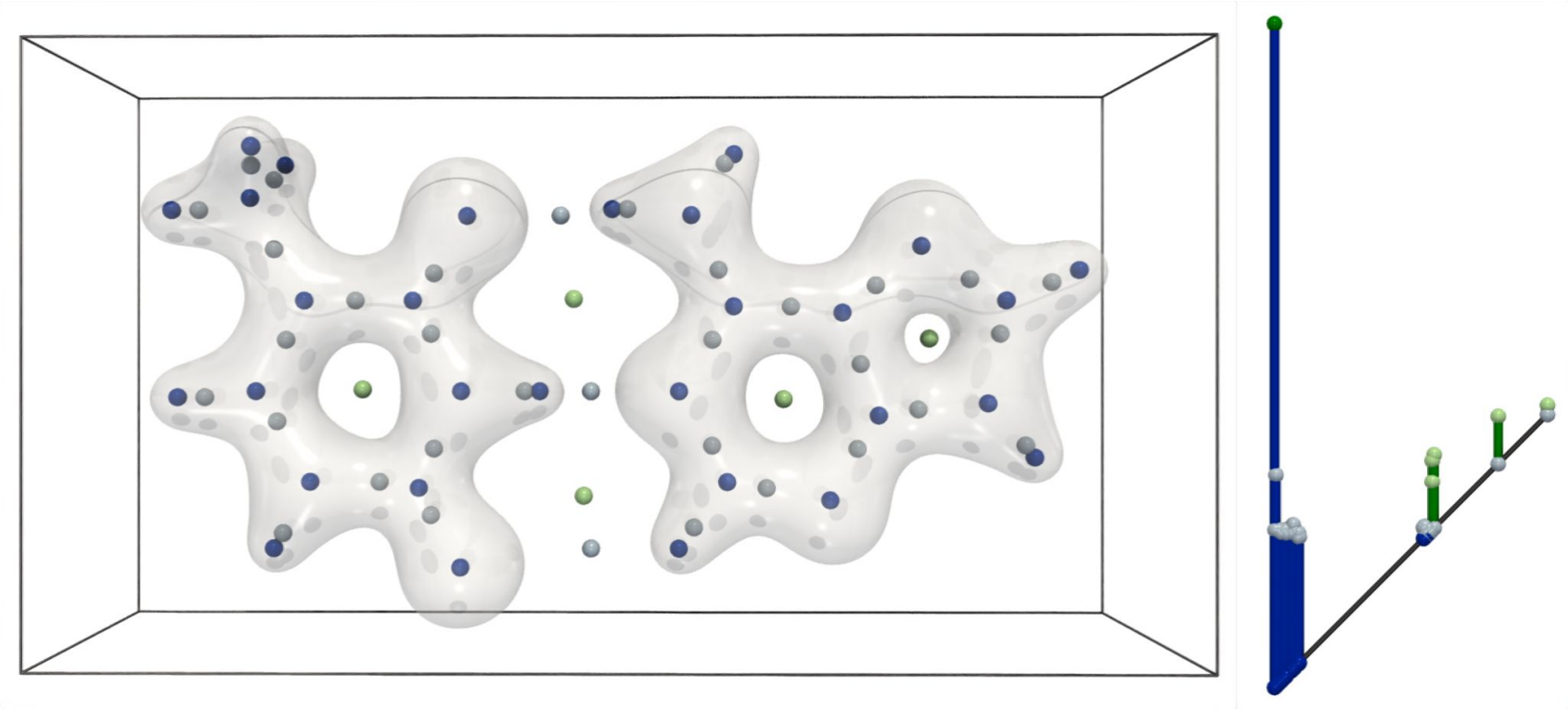
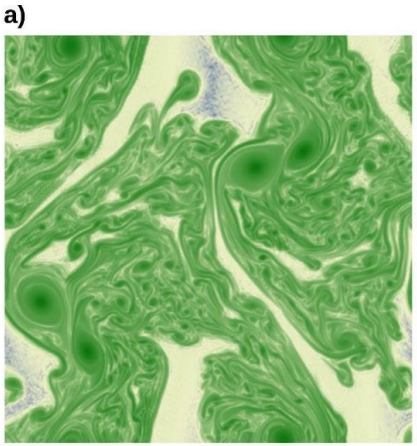


The persistence diagram

- **Structural features**
  - Vortex centers
    - Enstrophy maxima



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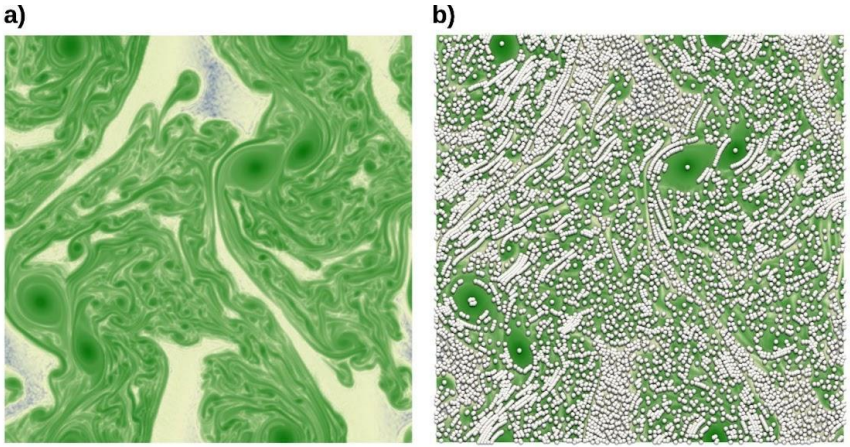

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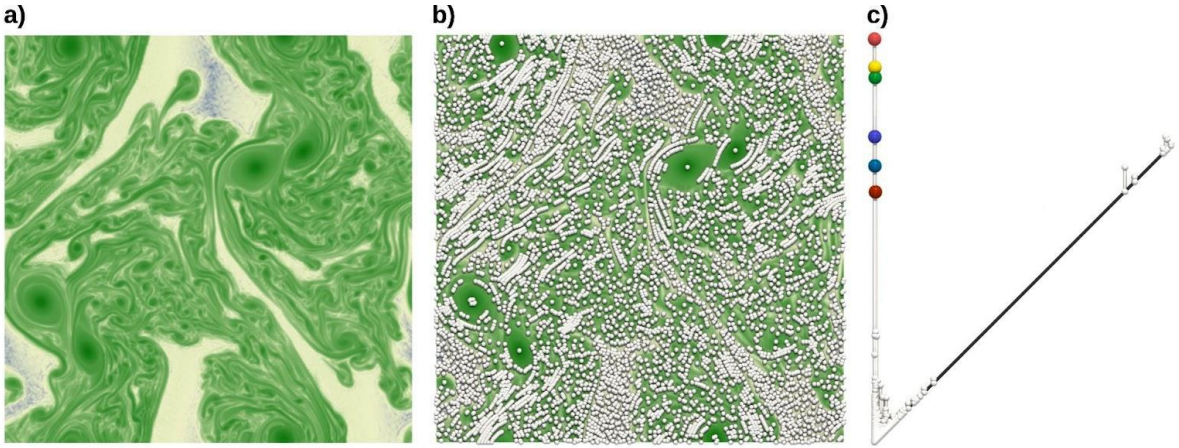

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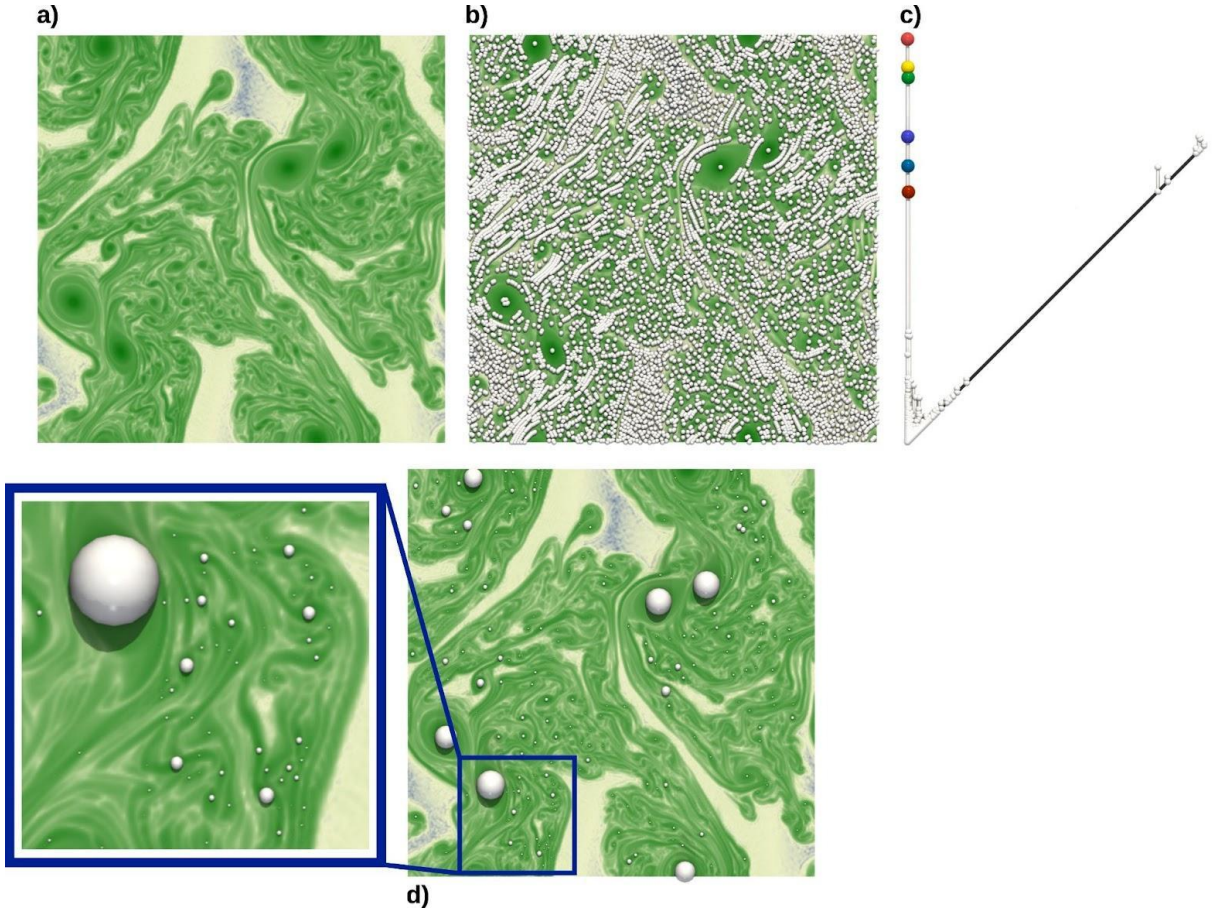

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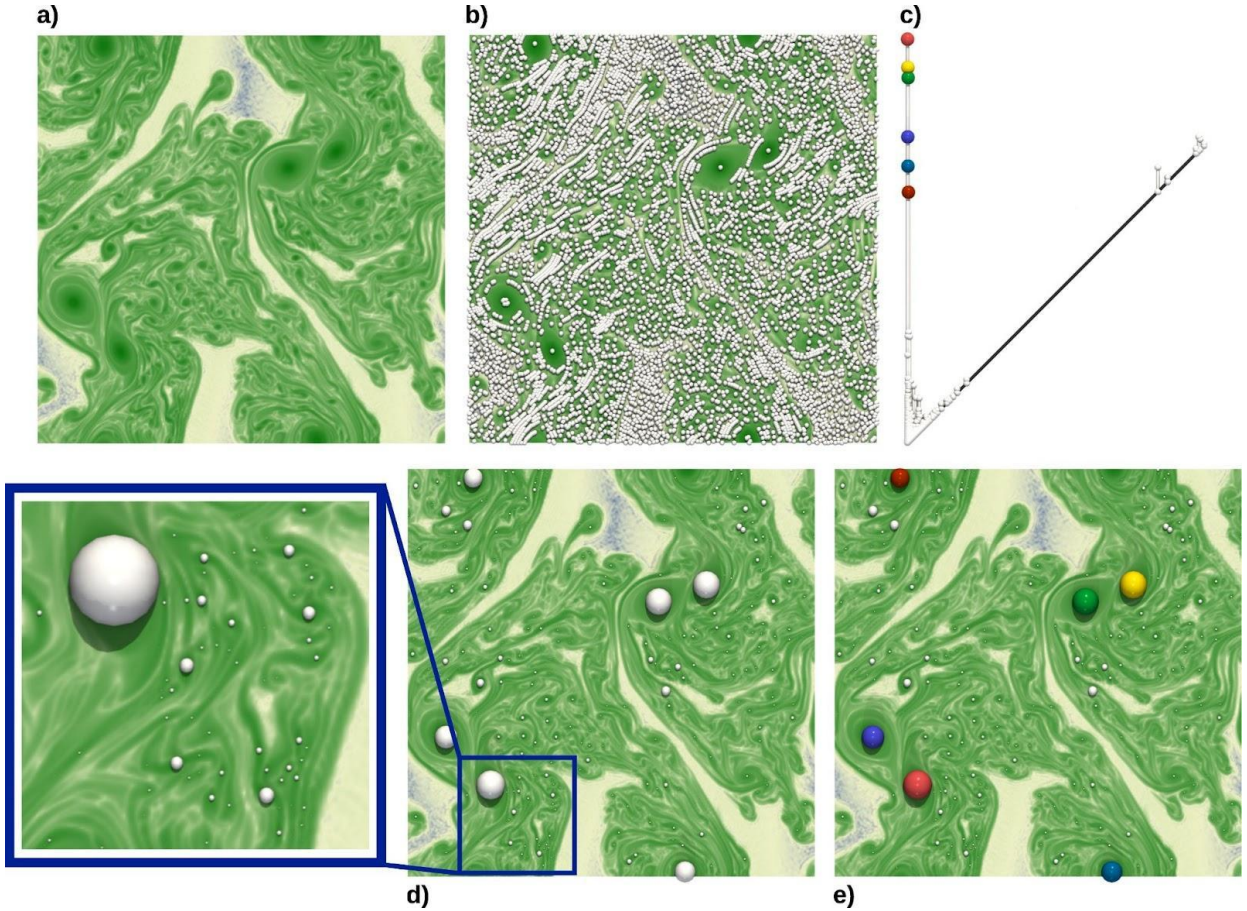
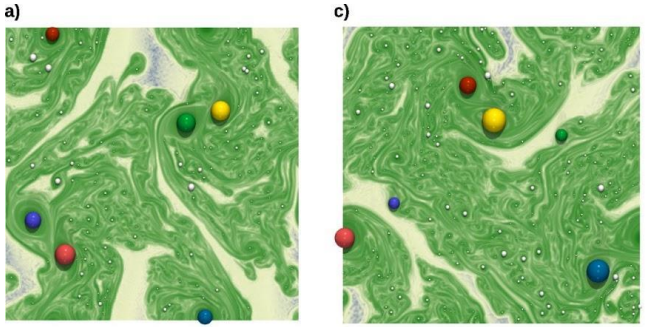


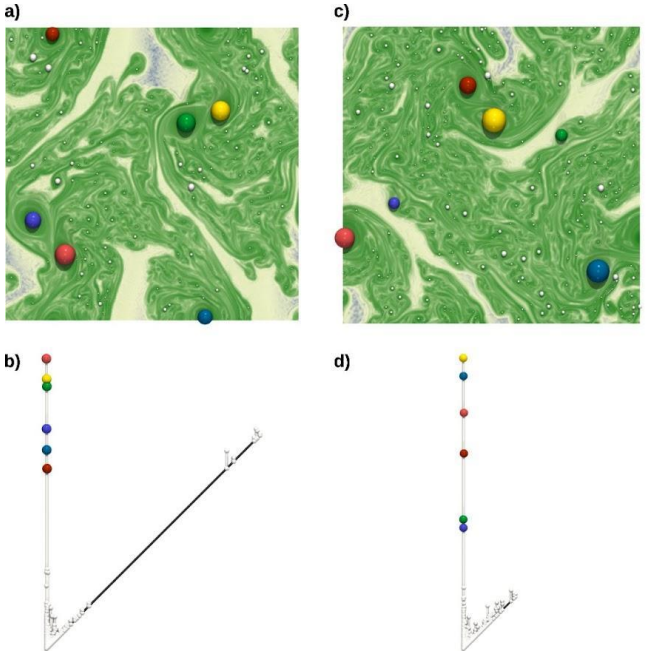
Comparing topologies

- Data representation



Comparing topologies

- **Data representation**
  - Topological signature
    - Persistence diagram



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    - Optimal assignment
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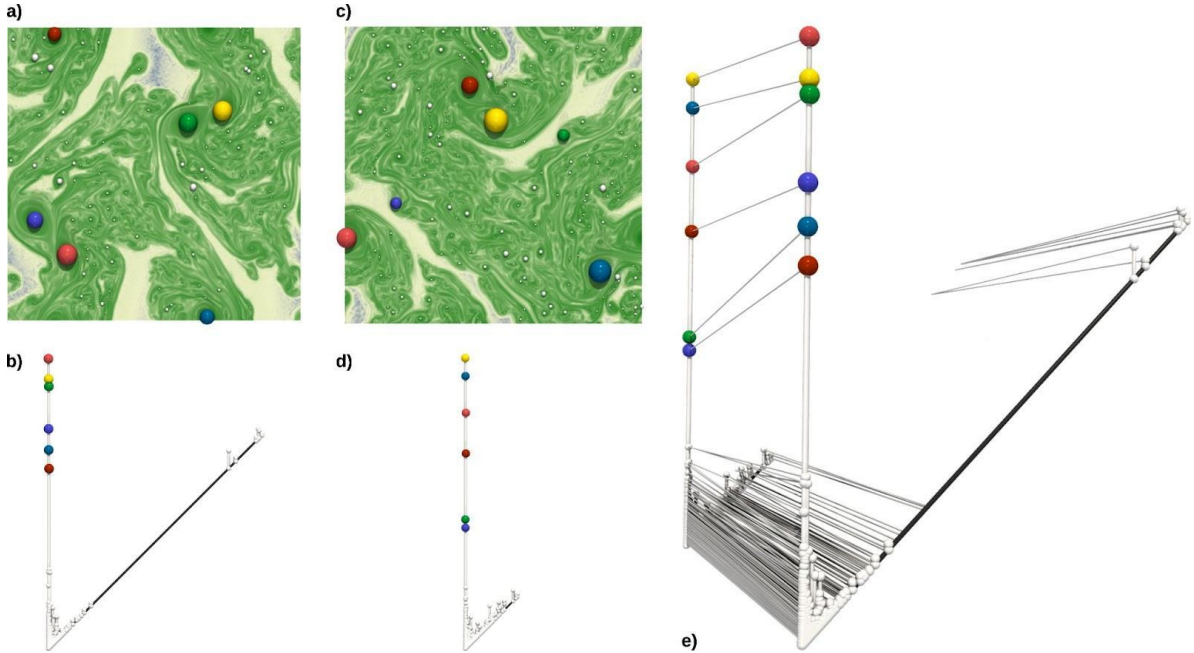

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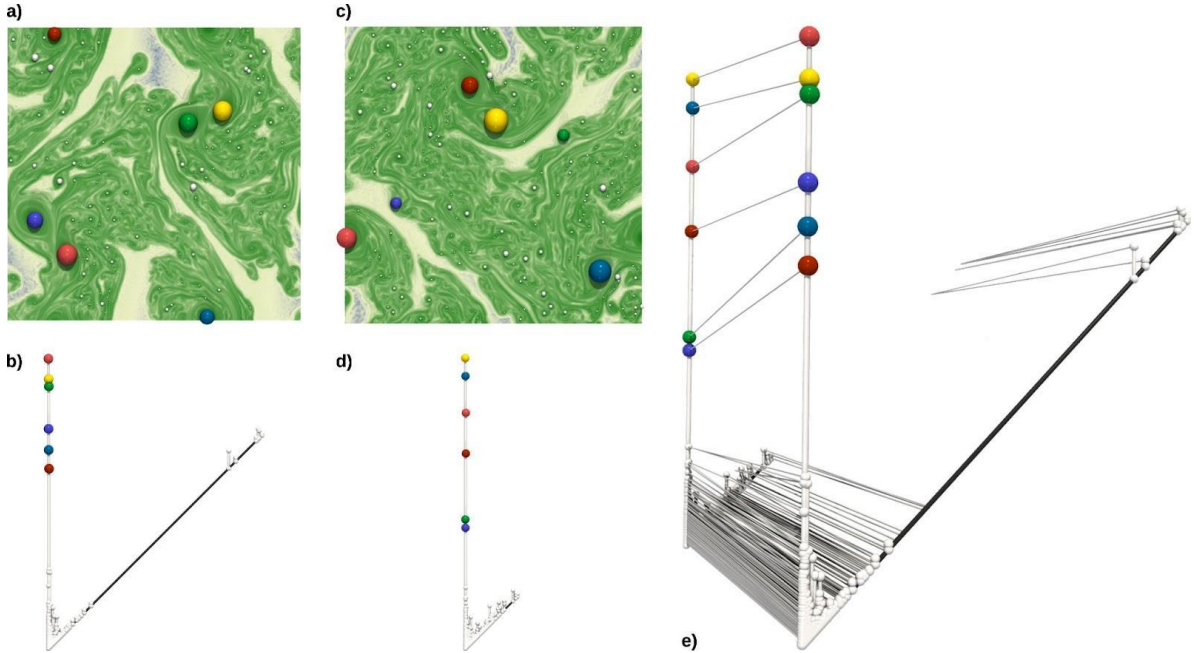
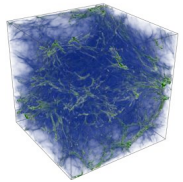
- ***Topological statistics***
  - Geodesics, means, …



The ERC project “TORI”



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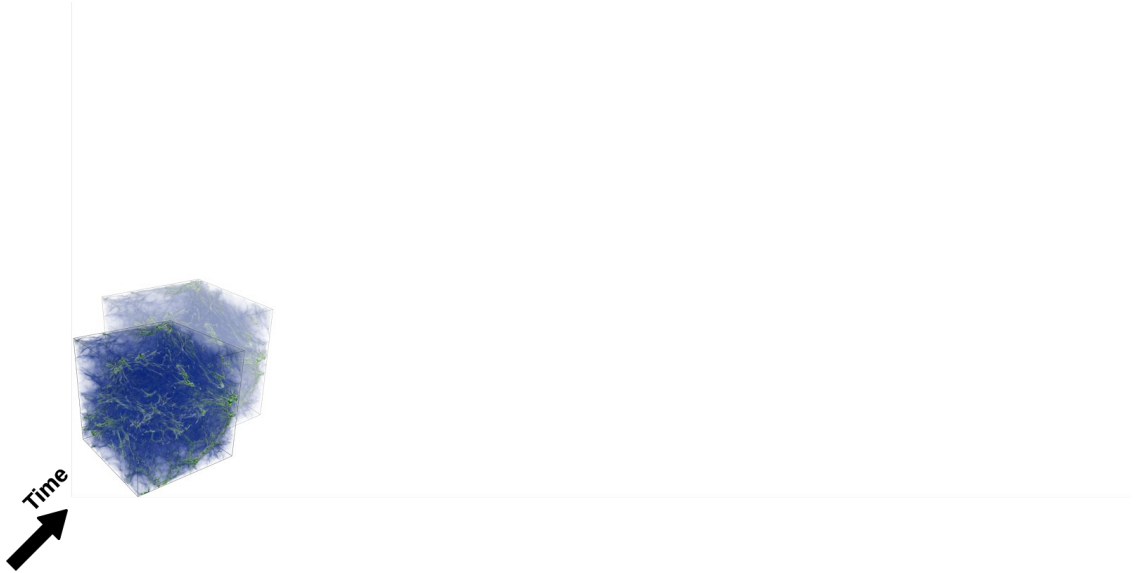

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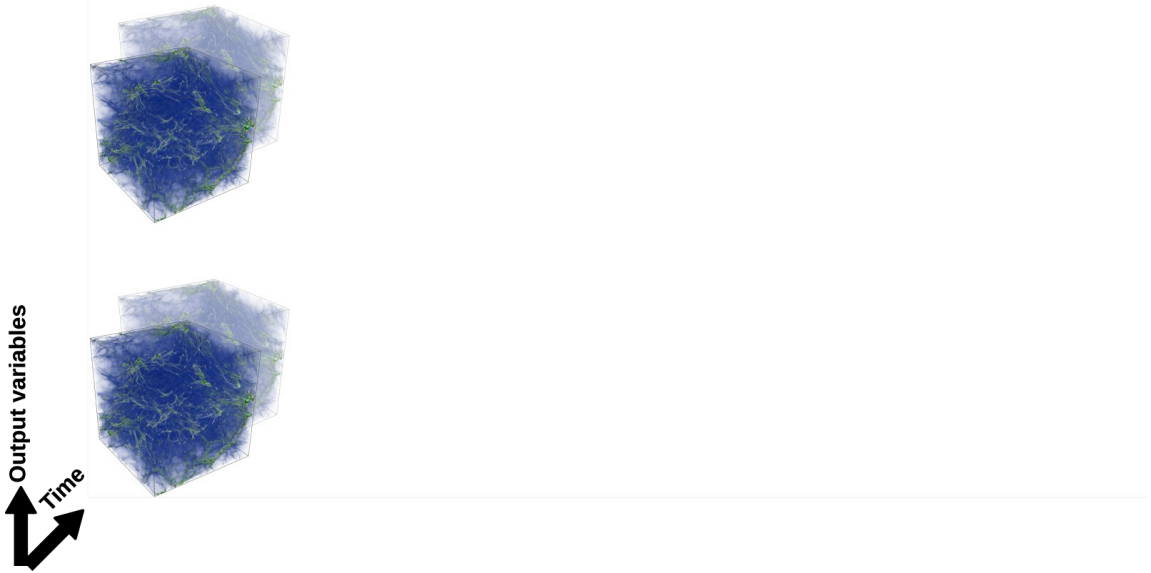

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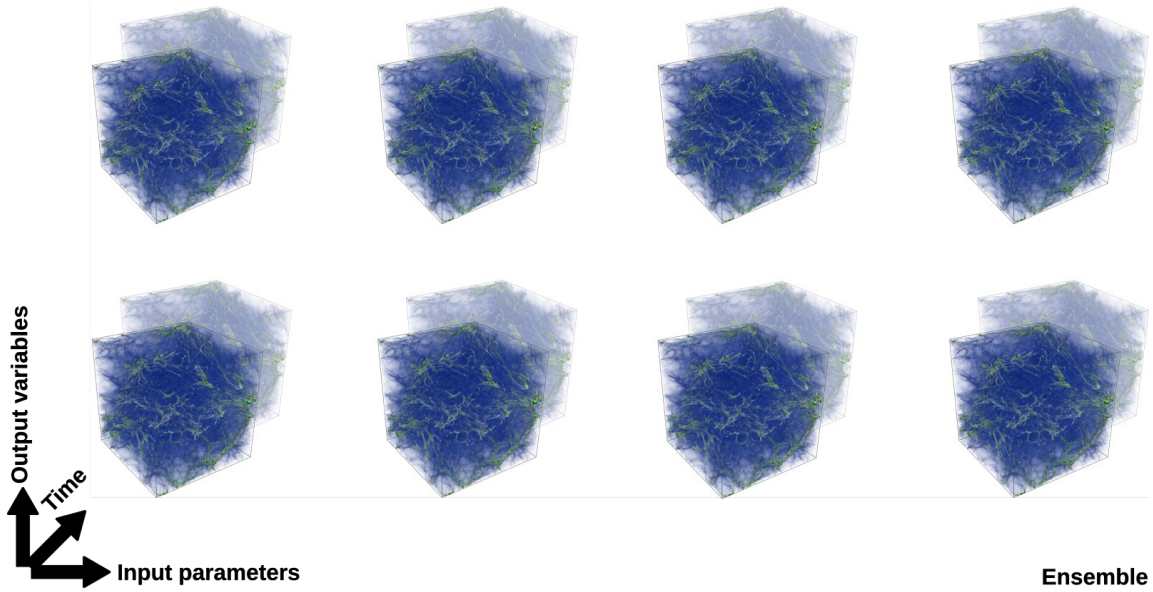

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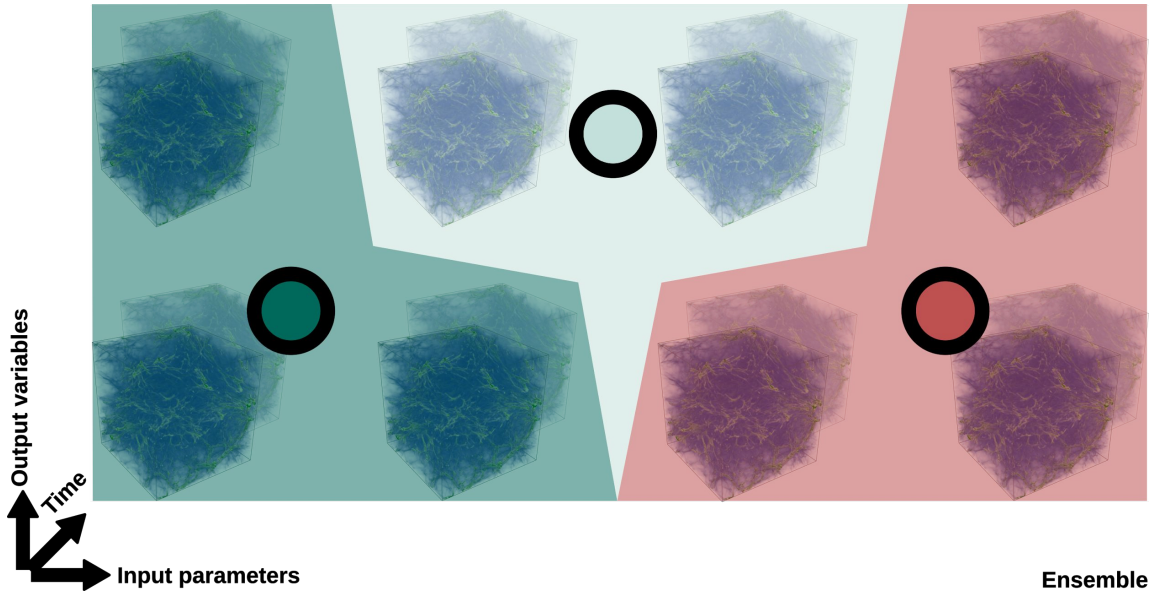

The ERC project “TORI”

- **Challenges**
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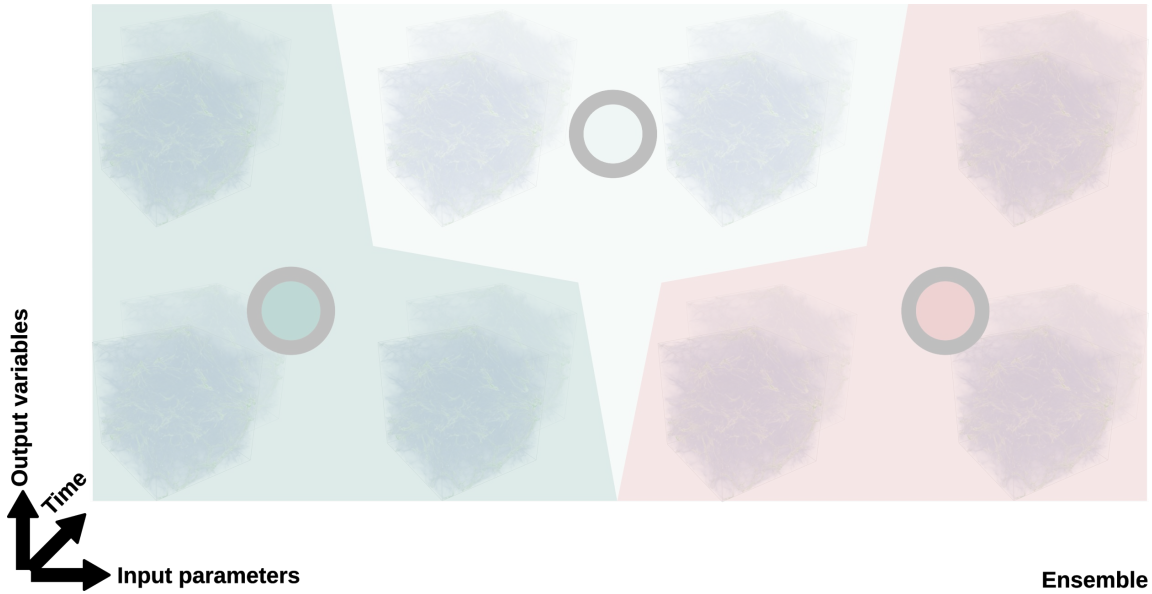
  - Data size



The ERC project “TORI”

- **Challenges**
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- **Strategy**
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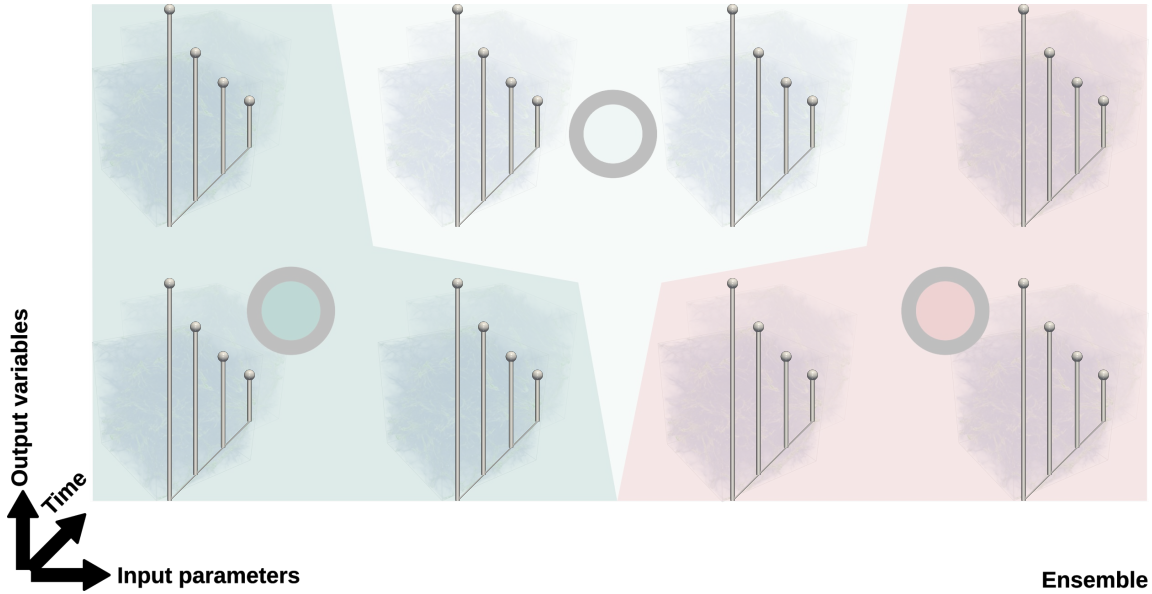

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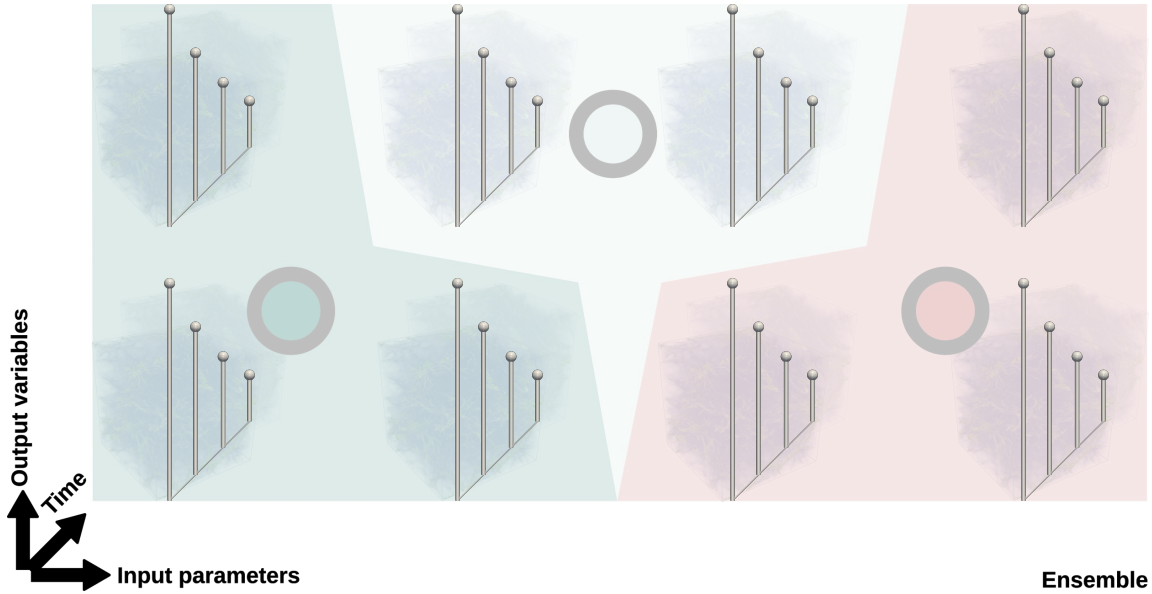
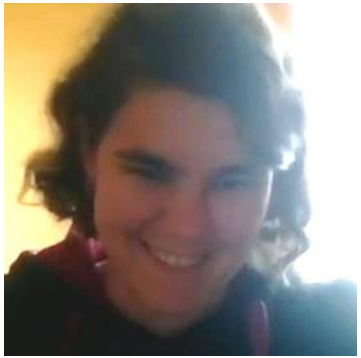


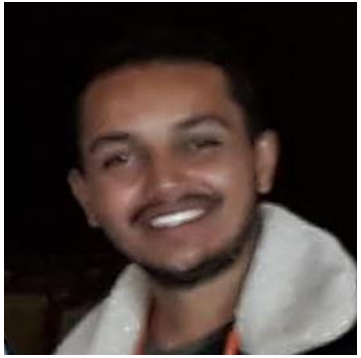
erc http://erc-tori.github.io/

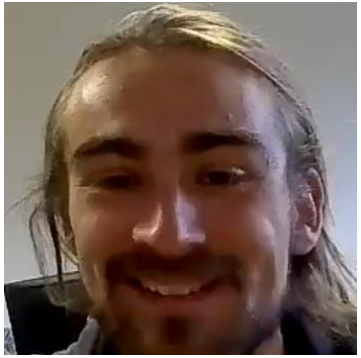
The TORI team





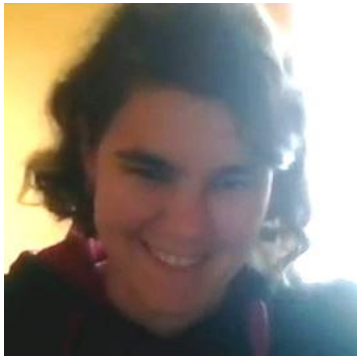




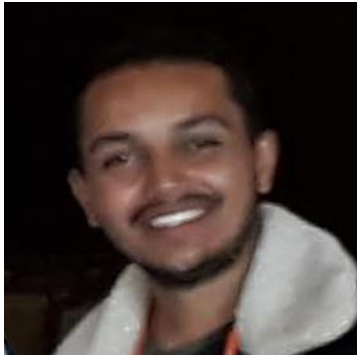
The TORI team

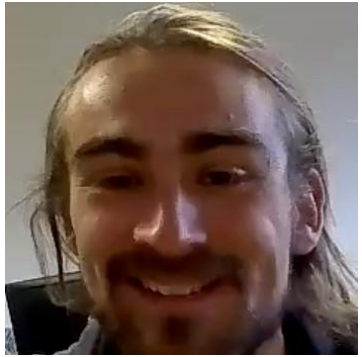
Topology at scale









The TORI team

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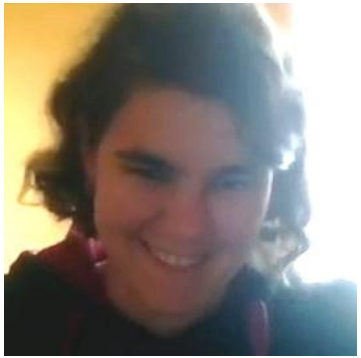
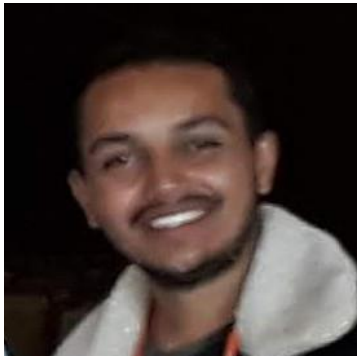




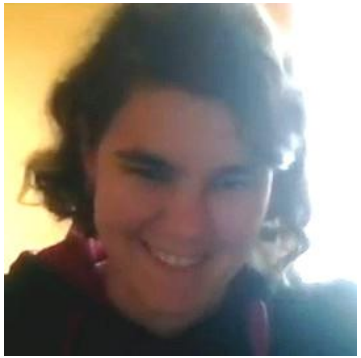



Topological statistics

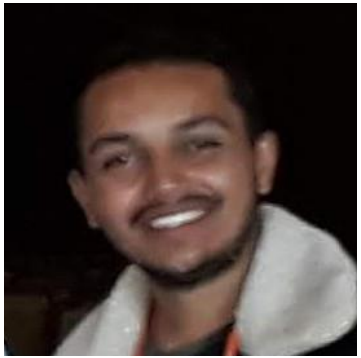
The TORI team

Topology at scale





Topological statistics

http://erc-tori.github.io/publications

Reproducible research

- Replicable papers
  - https://www.replicabilitystamp.org/

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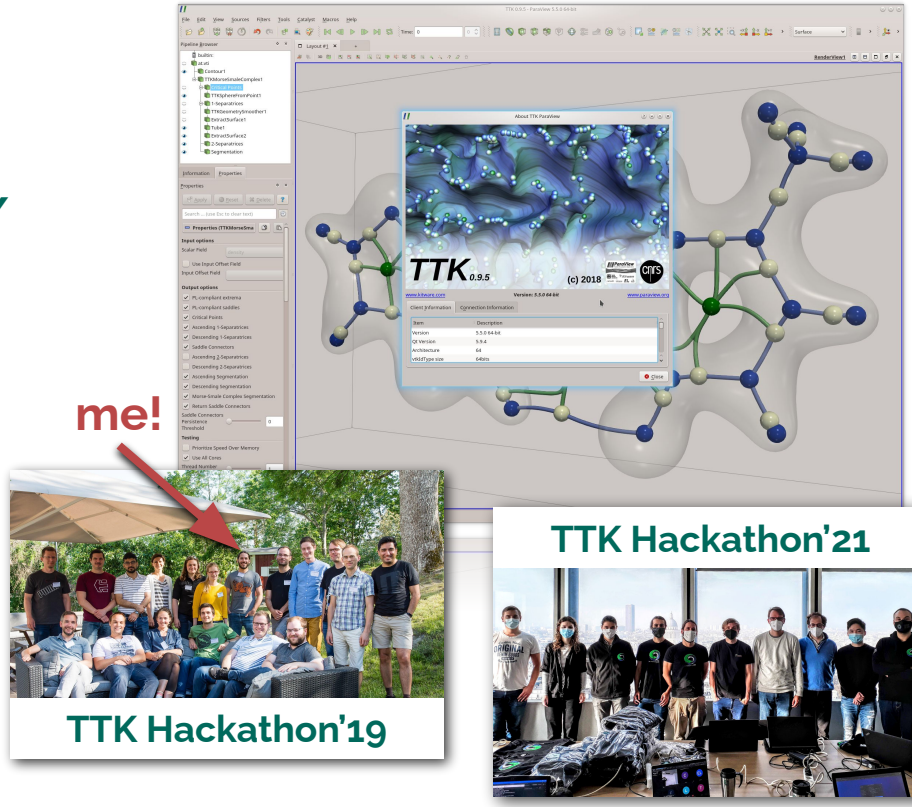
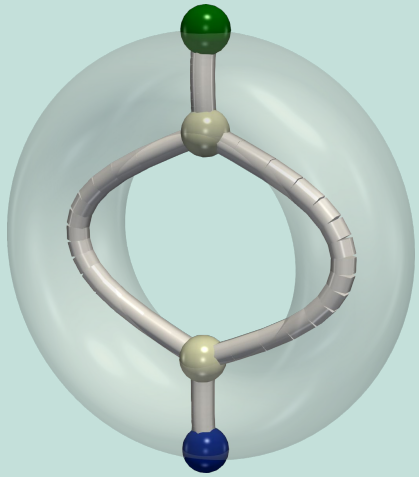
- **The Topology ToolKit (TTK)**
  - Open-source library C++ (2017)
    - **http://topology-tool-kit.github.io**
  - 17 contributing institutions
  - Online example database



Take-home messages

- **Numerical data?**
  - Recover/analyze/compare ‘structural’ information

  ⇨ Topological Data Analysis
    - Robust, multiscale
    - Successful in applications



Take-home messages

- **Numerical data?**
  - Recover/analyze/compare ‘structural’ information
  - ⇨ Topological Data Analysis
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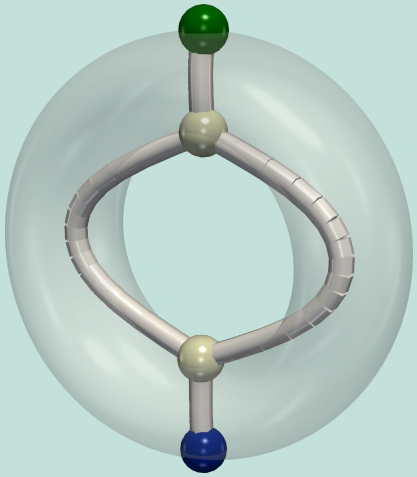
- **The TORI project**
  - Large scale data analysis
  - Available in open-source! (TTK)
  - Many perspectives



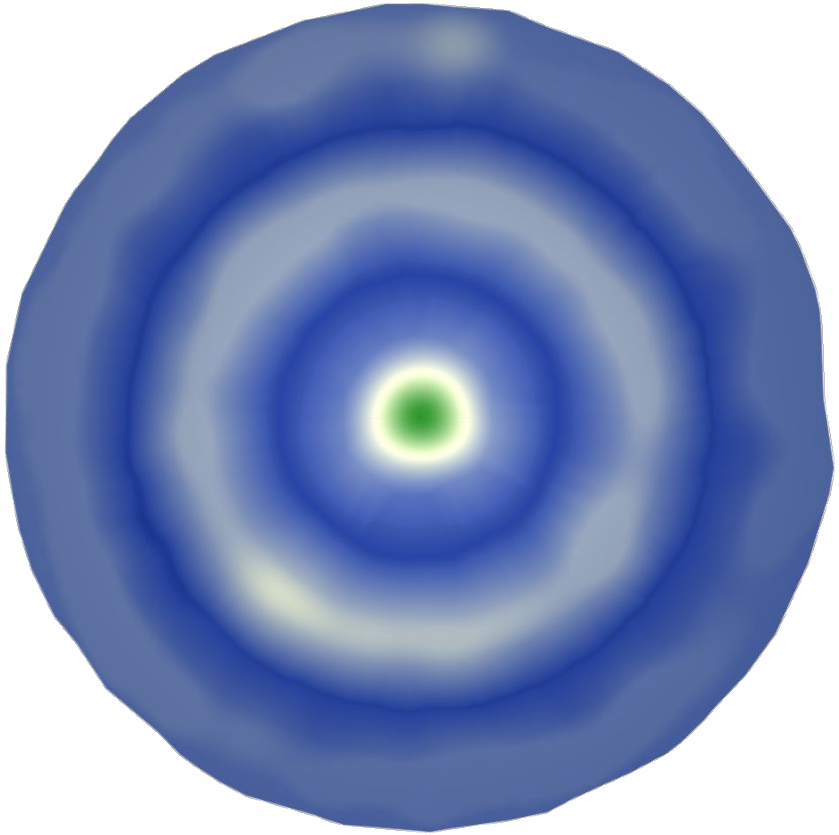
Thanks!

- **TTK**
  - Homepage: **https://topology-tool-kit.github.io/**
  - Examples: **https://topology-tool-kit.github.io/examples/**
  - Code: **https://github.com/topology-tool-kit/ttk/**

- **ERC Project TORI**
  - **https://erc-tori.github.io/**

The topology of numerical data

	Row ID	x	y
0	0	-0.565872	0.433185
1	1	-0.852855	-0.0414391
2	2	-0.74526	-0.0704809
3	3	-0.283754	0.732374
4	4	-0.348494	0.708251
5	5	-0.141337	-0.791217
6	6	-0.590461	0.429422
7	7	-0.611657	-0.551685
8	8	-0.601515	-0.45884
9	9	0.240081	0.697199
10	10	0.101971	0.836754
11	11	-0.706277	-0.36127
12	12	0.301446	-0.731123
13	13	-0.759347	0.297477
14	14	-0.685959	0.140645
15	15	-0.220021	-0.828495
16	16	0.794982	0.352219
17	17	0.61109	0.561273
18	18	-0.642173	0.313909
19	19	-0.769766	-0.0934766
20	20	-0.0163218	-0.845476



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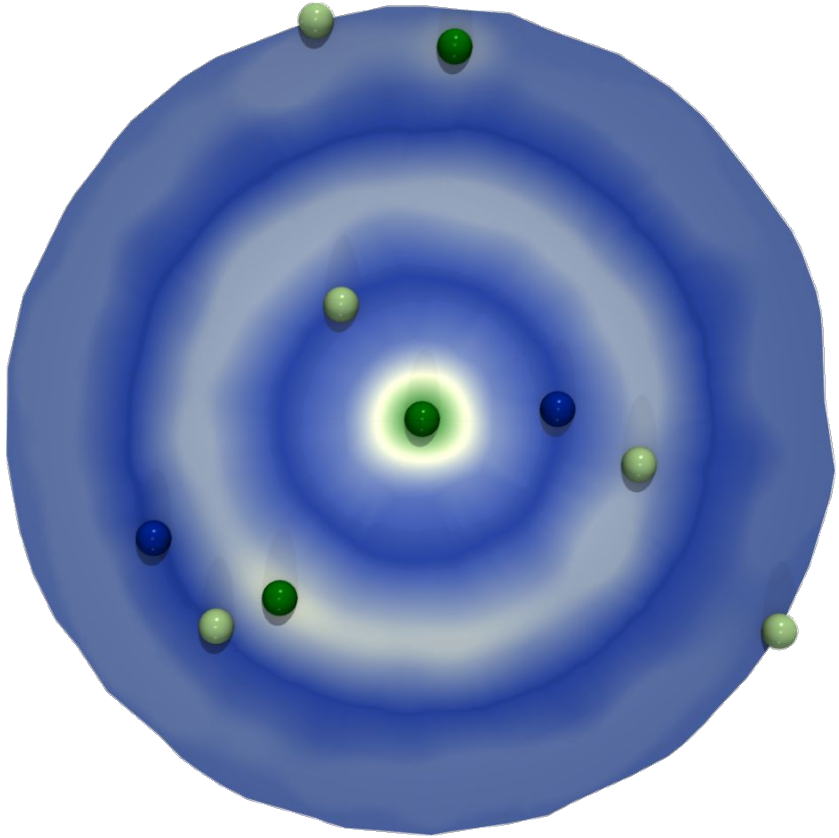

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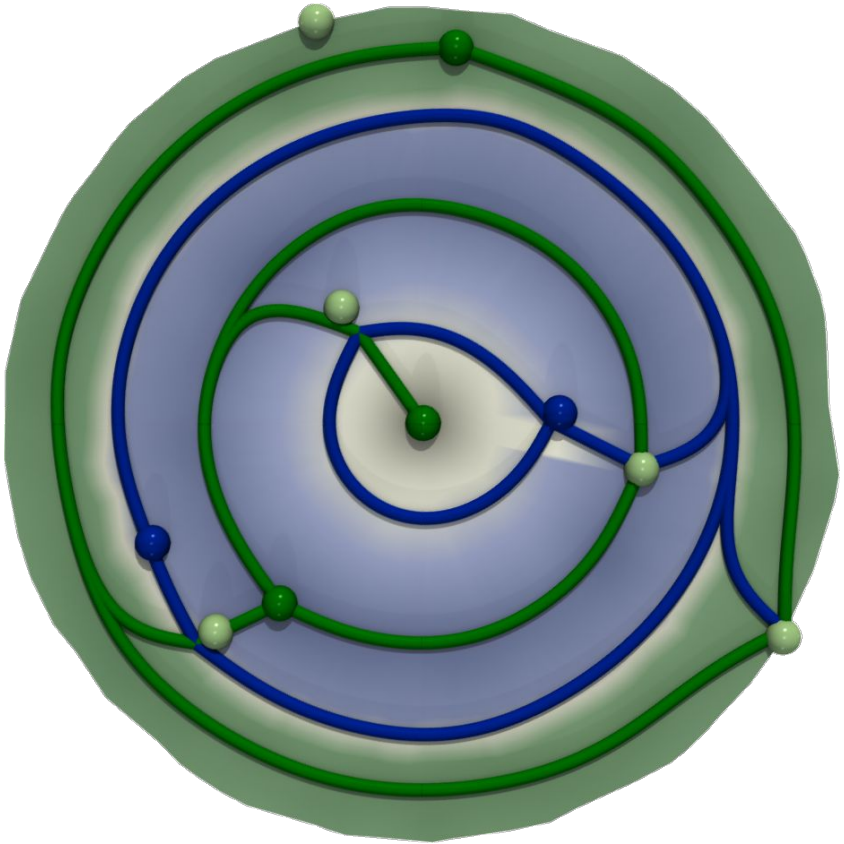

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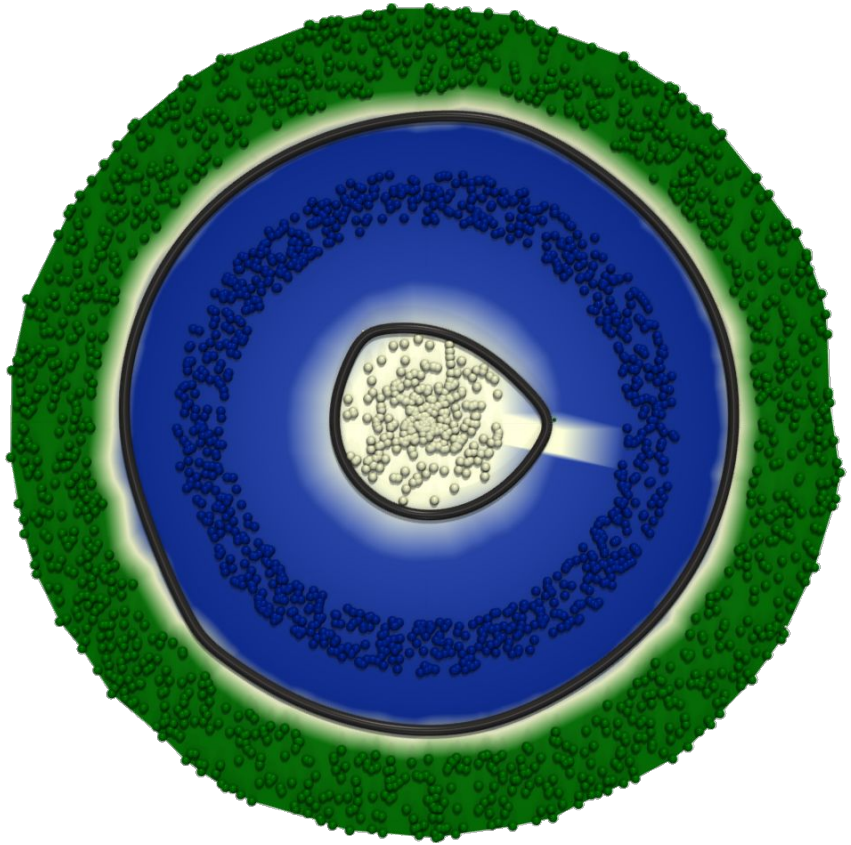

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