SISMA

Séminaire Informatique Scientifique & Mathématiques Appliquées

Harmonic potentials in the L2 de Rham complex

Martin Campos Pinto (Max Planck Society)

Being able to decompose vector fields in terms of scalar and vector potentials is a central tool in physical and computational modelling. In contractible domains this corresponds to the Helmholtz decomposition into irrotational (curl-free) and solenoidal (divergence-free) fields.
Domains with holes, however, admit non-trivial harmonic fields which are both curl- and divergence-free but do not necessarily admit scalar or vector potentials. Specifically, 3D holes can be of cavity or tunnel type, corresponding to the 2-chain and 1-chain homology groups. For harmonic fields normal to the boundary, which exist in domains with cavities, the standard approach constructs scalar potentials by solving Laplace's equation with Dirichlet boundary conditions fitted to the domain cavities. For tangent harmonic fields, which exist in domains with tunnels, a similar method was missing: a construction of piecewise smooth scalar potentials does exist, but it is limited to domains which can be made contractible by removing a collection of disjoint “cutting surfaces”, and it does not define proper vector potentials.

In this talk I will present a constructive geometric parametrisation of tangent harmonic fields by vector potentials associated to the tunnels of a 3D domain. Because our analysis relies on reciprocal surfaces which are in intersection duality with the tunnel curves, our method does not assume restrictive topological constraints: the existence of reciprocal surfaces in general domains follows from the Poincaré-Lefschetz duality.

Finally, a natural application of our construction to structure-preserving finite elements provides an exact geometric parametrisation of the discrete tangent harmonic fields in terms of discrete potentials.

This is a joint work with Julian Owezarek (TUM).

Contact: lihpc-contact@cea.fr