SISMA

Séminaire Informatique Scientifique & Mathématiques Appliquées

Numerical analysis of lattice Boltzmann schemes: general results and monotonicity/convergence for a non-linear conservation law

Thomas Bellotti

Université Paris-Saclay, CNRS, CentraleSupélec

Lattice Boltzmann schemes are very efficient explicit numerical methods to solve problems under the form of conservation laws, which increase the number of unknowns in the problem. However, partly due to this reason, they suffer from a chronic lack of clear theoretical foundations.

I will first quickly recapitulate some key results on the numerical analysis of lattice Boltzmann schemes (consistency, stability, and well-preparation of initial data) on infinite/periodic domains, which are mainly based on the algebraic elimination of the additional unknowns. On this occasion, I will also hint at research paths and related issues concerning boundary conditions.

Then, I will dig with more details into a proof of convergence for a broad family of lattice Boltzmann schemes with two relaxation times, whose interest I will justify, towards the weak entropy solution of a multidimensional conservation law. To this end, we adapt the compactness-based proof technique of Crandall and Majda, which has become "classical" for scalar finite volume schemes, and is based on monotonicity properties of the schemes. Two key points in the proof are, on the one hand, a result on the monotonicity of equilibria (Maxwellians), which enables the use of kinetic Kruzhkov entropies, and, on the other hand, a result on the return to equilibrium of the discrete solution. I will conclude the presentation with numerical illustrations that support our theoretical results.