SISMA

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A geometrically and thermodynamically compatible finite volume scheme for continuum mechanics on unstructured polygonal meshes

Raphaël Loubère
joint work with Walter Boscheri and Pierre-Henri Maire
Institut de Mathématiques de Bordeaux

We present a novel Eulerian Finite Volume (FV) scheme on unstructured polygonal meshes that is provably compliant with the Second Law of Thermodynamics and the Geometric Conservation Law (GCL) at the same time. The governing equations are provided by a subset of the class of symmetric and hyperbolic thermodynamically compatible (SHTC) models introduced by Godunov in 1961. Specifically, our numerical method discretizes the equations for the conservation of momentum, total energy, distortion tensor and thermal impulse vector, hence accounting in one single unified mathematical formalism for a wide range of physical phenomena in continuum mechanics, spanning from ideal and viscous fluids to hyperelastic solids. By means of two conservative corrections directly embedded in the definition of the numerical fluxes, the new schemes are proven to satisfy two extra conservation laws, namely an entropy balance law and a geometric equation that links the distortion tensor to the density evolution. As such, the classical mass conservation equation can be discarded. Firstly, the GCL is derived at the continuous level, and subsequently it is satisfied by introducing the new concepts of general potential and generalized Gibbs relation. The new potential is nothing but the determinant of the distortion tensor, and the associated Gibbs relation is derived by introducing a set of dual or thermodynamic variables such that the GCL is retrieved by dot multiplying the original system with the new dual variables. Once compatibility of the GCL is ensured, thermodynamic compatibility is tackled in the same manner, thus achieving the satisfaction of a local cell entropy inequality. The two corrections are orthogonal, meaning that they can coexist simultaneously without interfering with each other. The compatibility of the new FV schemes holds true at the semi-discrete level, and time integration of the governing PDE is carried out relying on Runge-Kutta schemes. A large suite of test cases demonstrates the structure preserving properties of the schemes at the discrete level as well.

Contact: lihpc-contact@cea.fr