SISMA

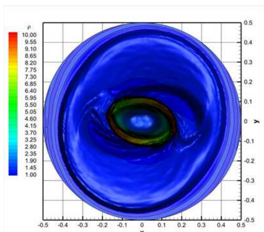
Séminaire Informatique Scientifique & Mathématiques Appliquées

An unconventional divergence preserving Finite Volume discretization of Lagrangian ideal MHD

W. Boscheri (CNRS, U. Savoie Mt-Blanc), Raphaël Loubère (CNRS, U. Bordeaux) & P.-H. Maire (CESTA)

In this work we describe the construction of an unconventional divergence preserving discretization of updated Lagrangian ideal MHD over simplicial grids. The cell-centered Finite Volume method employed to discretize the conservation laws of volume, momentum and total energy is rigorously the same than the one developed to simulate hyperelasticity equations [1]. By construction this moving mesh method ensures the compatibility between the mesh displacement and the approximation of the volume flux by means of the nodal velocity and the attached unit corner normal vector which is nothing but the partial derivative of the cell volume with respect to the node coordinate under consideration. This is precisely the definition of the compatibility with the Geometrical Conservation Law which is the cornerstone of any proper multi-dimensional moving mesh FV discretization. The momentum and the total energy fluxes are approximated utilizing the partition of cell faces into sub-faces and the concept of sub-face force which is the traction force attached to each sub-face impinging at a node. We observe that the time evolution of the magnetic field might be simply expressed in terms of the deformation gradient which characterizes the Lagrange-to-Euler mapping. In this framework the divergence of the magnetic field is conserved with respect to time thanks to the Piola formula. Therefore, we solve the fully compatible updated Lagrangian discretization of the deformation gradient tensor for updating in a simple manner the cell-centered value of the magnetic field [2]. Finally, the sub-face traction force is expressed in terms of the nodal velocity to ensure a semi-discrete entropy inequality within each cell. The conservation of momentum and total energy is recovered prescribing the balance of all the sub-face forces attached to the sub-faces impinging at a given node. This balance corresponds to a vectorial system satisfied by the nodal velocity. It always admits a unique solution which provides the nodal velocity. The robustness and the accuracy of this unconventional FV scheme has been demonstrated employing various representative test cases. Finally, it is worth emphasizing that once you have an updated Lagrangian code for solving hyperelasticity you also get an updated Lagrangian code for solving ideal MHD preserving exactly the divergence of the magnetic field at the discrete level.

References :
[1] Boscheri, W., Loubère, R., Maire, P.-H.: A 3D cell-centered ADER MOOD Finite Volume method for solving updated Lagrangian hyperelasticity on unstructured grids. Journal of Computational Physics 449 (2022).
[2] Boscheri, W., Loubère, R., Maire, P.-H.: An unconventional divergence preserving Finite Volume discretization of Lagrangian ideal MHD. Communications on Applied Mathematics and Computation (2023).



Contact: lihpc-sisma@cea.fr